Workshop on Combinatorics and Graph Theory

21th, April, 2006 · Nankai University

Single machine rescheduling problem under a limit disruption of jobs

Yuan Jinjiang

Department of mathematics, Zhengzhou University
Zhengzhou, Henan 450052

yuanjj@zzu.edu.cn

1 Introduction and Problem Formulation

• Rescheduling, just as its name implies, means to schedule the jobs again, together with a set of new jobs.

• In the rescheduling on a single machine, a set of original jobs has already been scheduled to minimize some cost objective, when a new set of jobs arrives and creates a disruption. The decision maker needs to insert the new jobs into the existing schedule without excessively disrupting it.

• By Hall and Potts (2004), the rescheduling problem for jobs on a single machine can be stated as follows.

Let $\mathcal{J}_O = \{J_1, ..., J_{n_O}\}$ denote a set of original jobs to be processed non-preemptively on a single machine.

In the model, we assume that the jobs in $\mathcal{J}_O$ have been scheduled optimally to minimize some classical objective and that π^* is an optimal schedule.

Let $\mathcal{J}_N = \{J_{n_O+1}, ..., J_{n_O+n_N}\}$ denote a set of new jobs.

Write $\mathcal{J} = \mathcal{J}_O \cup \mathcal{J}_N$.

Each job $J_j \in \mathcal{J}$ has an integral processing time $p_j \geq 0$, an integer release date $r_j \geq 0$ and an integer due date d_j.

We assume that the new jobs' information (processing times and release dates) becomes known at time zero after a schedule for the jobs of J_O has been determined, but before processing begins.

Let $n = n_O + n_N$.

Let π^* and σ^* denote an optimal schedule of the jobs of $\mathcal{J}_O$ and $\mathcal{J}$, respectively.

- •For any schedule σ of the jobs in $\mathcal{J}$, we define the following variables:
- $S_j(\sigma)$ is the time at which job $J_j \in \mathcal{J}$ starts its processing.
- $C_j(\sigma) = S_j(\sigma) + p_j$ is the time at which job $J_j \in \mathcal{J}$ is completed.
- $C_{\max}(\sigma) = \max\{C_j(\sigma)\}$ is the makespan of jobs in $\mathcal{J}$ under the schedule σ.
- $D_j(\pi^*, \sigma)$ is the sequence disruption of job $J_j \in \mathcal{J}_O$, i.e., if J_j is the x-th job in π^* and the y-th job in σ, respectively, then $D_j(\pi^*, \sigma) = |y - x|$.
- $\Delta_j(\pi^*, \sigma) = |C_j(\sigma) - C_j(\pi^*)|$ is the time disruption of job $J_j \in \mathcal{J}_O$.

  Here the sequence disruption of job $J_j \in \mathcal{J}_O$ in schedule σ is the absolute value of the difference between the positions of that job in σ and π^*.

  When there is no ambiguity, the above five parameters are simplified to C_j, $C_{\max}$, $D_j(\pi^*)$, and $\Delta_j(\pi^*)$, respectively.

- The standard classification scheme for scheduling problems is a three-field classification $\alpha|\beta|\gamma$, where α indicates the scheduling environment, β describes the job characteristics or restrictive requirements, and γ defines the optimality criterion.

  Here we consider only single-machine problems, thus implying that $\alpha = 1$.

  Under β, we indicate a constraint on the amount of disruption where applicable.

  Such constraints include the following four forms:

- $D_{\max}(\pi^*) \leq k$: $\max_{J_j \in \mathcal{J}_O}\{D_j(\pi^*)\} \leq k$, the maximum sequence disruption of the jobs cannot exceed k.

- $\sum D_j(\pi^*) \leq k$: $\sum_{J_j \in \mathcal{J}_O} D_j(\pi^*) \leq k$, the total sequence disruption of the jobs cannot exceed k.

- $\Delta_{\max}(\pi^*) \leq k$: $\max_{J_j \in \mathcal{J}_O}\{\Delta_j(\pi^*)\} \leq k$, the maximum time disruption of the jobs cannot exceed k.

- $\sum \Delta_j(\pi^*) \leq k$: $\sum_{J_j \in \mathcal{J}_O} \Delta_j(\pi^*) \leq k$, the total time disruption of the jobs cannot exceed k.

Let f be the cost function to be minimized.

The scheduling problems are of the following forms:

- $1|D_{\max}(\pi^*) \leq k|f$
- $1|\sum D_j(\pi^*) \leq k|f$
- $1|\Delta_{\max}(\pi^*) \leq k|f$
- $1|\sum \Delta_j(\pi^*) \leq k|f$.

For a job set $\mathcal{E} \subseteq \mathcal{J}$, a schedule σ of the jobs in $\mathcal{E} \subseteq \mathcal{J}$ is called regular for $\mathcal{E}$ if there is no other schedule h such that $C_j(h) \leq C_j(\sigma)$ for every job J_j and there is at lease one job J_i such that $C_i(h) < C_i(\sigma)$.

The jobs in a regular schedule are said to be regularly scheduled.

•• We will only consider optimal schedules, since there must be an optimal schedule which is regular when $f = f(C_1, ..., C_n)$ is non-decreasing for each C_j.

2 The existing complexity results and open problems

- Polynomially solved problems:

$1\|\Delta_{\max}(\pi^*) \leq k\|L_{\max}$,	$O(n + n_N \log n_N)$,	Hall and Potts (OR, 2004)
$1\|D_{\max}(\pi^*) \leq k\|L_{\max}$,	$O(n + n_N \log n_N)$,	Hall and Potts (OR, 2004)
$1\|\Delta_{\max}(\pi^*) \leq k\|\sum C_j$,	$O(n + n_N \log n_N)$,	Hall and Potts (OR, 2004)
$1\|D_{\max}(\pi^*) \leq k\|\sum C_j$,	$O(n + n_N \log n_N)$,	Hall and Potts (OR, 2004)
$1\|\sum D_j(\pi^*) \leq k\|\sum C_j$,	$O(n_O^2 n_N^2)$,	Hall and Potts (OR, 2004)
$1\|r_j, D_{\max}(\pi^*) \leq k\|C_{\max}$,	$O(n_N^2(n_O + n_N))$,	Yuan and Mu (2006)

• NP-hard problems:

$1\|\sum \Delta_j(\pi^*) \le k\|L_{\max}$,	strongly NP-hard,	Hall and Potts (OR, 2004)
$1\|\sum \Delta_j(\pi^*) \le k\|\sum C_j$,	$O(n_O n_N \min\{n_O P_N, n_N P_O\})$,	Hall and Potts (OR, 2004)
$1\|r_j, \Delta_{\max}(\pi^*) \le k\|C_{\max}$,	strongly NP-hard,	Yuan and Mu (2006)
$1\|r_j, \sum \Delta_j(\pi^*) \le k\|C_{\max}$,	strongly NP-hard,	Yuan and Mu (2006)

• Open problems:

(1) $1|\sum D_j(\pi^*) \le k|L_{\max}$. See Hall and Potts (OR, 2004).

(2) $1|r_j, \sum D_j(\pi^*) \le k|C_{\max}$. Yuan and Mu (2006).

3 $1|r_j, D_{\max}(\pi^*) \le k|C_{\max}$

The following classical scheduling rule will be used.

• **ERD Rule:** In the earliest release date (ERD) rule, jobs are regularly sequenced in non-decreasing order of their release dates.

• It is well-known that the ERD rule provides an optimal schedule for the classical scheduling problem $1|r_j|C_{\max}$.

• We always assume that the jobs of $\mathcal{J}_O$ are indexed and sequenced in ERD order in π^*, i.e.,

$$r_1 \le r_2 \le ... \le r_{n_O}.$$

• For simplicity, we also assume that the new jobs are indexed in ERD order, i.e.,

$$r_{n_O+1} \le r_{n_O+2} \le ... \le r_{n_O+n_N}.$$

• For a job set $\mathcal{E} \subseteq \mathcal{J}$, the regular schedule of the jobs in $\mathcal{E}$ obtained by ERD rule is called a regular ERD schedule of $\mathcal{E}$. Such a schedule is denoted by $ERD(\mathcal{E})$.

• We use $C(\mathcal{E})$ to denote the makespan of the schedule $ERD(\mathcal{E})$, i.e.,

$$C(\mathcal{E}) = C_{\max}(ERD(\mathcal{E})).$$

• A benefit of a regular ERD schedule $ERD(\mathcal{E})$ is that, for every job $J_j \in \mathcal{E}$, there are no idle times between r_j and S_j (the starting time of J_j) in the schedule $ERD(\mathcal{E})$.

(Let S_j be the starting time of J_j in a regular ERD schedule σ. If there are idle times between r_j and S_j, there must be an idle-time interval $[x, y)$ such that $r_j < y \le S_j$. Let J_i be the first job in σ such that $S_i \ge y$. Then $S_i \le S_j$ and the machine is idle in the time interval $[x, S_i)$. Since σ is a regular ERD schedule, we have $r_i \le r_j < y$. By shifting job J_i left with starting at time $\max\{x, r_i\}$, we obtain another schedule h with $C_i(h) < C_i(\sigma)$ and $C_l(h) = C_l(\sigma)$ for $l \ne i$. This contradicts the assumption that σ is a regular schedule. Hence, there is no idle time between r_j and S_j.) This fact will be used in the next section.

- Write $\mathcal{J}_M = \{J_j \in \mathcal{J}_N : r_j < r_{n_O}\}$.

Lemma 1 The problem $1|r_j, D_{\max}(\pi^*) \leq k|C_{\max}$ under the job system $(\mathcal{J}_O, \mathcal{J}_N)$ can be polynomially reduced to the corresponding problem under the job system $(\mathcal{J}_O, \mathcal{J}_M)$.

Proof We can observed that, there is an optimal schedule for $1|r_j, D_{\max}(\pi^*) \leq k|C_{\max}$ under the job system $(\mathcal{J}_O, \mathcal{J}_N)$ such that the new jobs with release dates greater than or equal to r_{n_O} (the largest release date of the original jobs) are scheduled at the end of the schedule and be ordered in the ERD order. Hence, the essential problem is to determine the minimum makespan of the jobs in $(\mathcal{J}_O, \mathcal{J}_M)$ under the limit of maximum sequence disruption. The result follows. □

• Based on Lemma 1, we always assume in the following that $\mathcal{J}_N = \mathcal{J}_M$. This means that for each new job $J_j \in \mathcal{J}_N$ we have $r_j < r_{n_O}$.

• For a schedule π of $\mathcal{J} = \mathcal{J}_O \cup \mathcal{J}_N$ and a job $J_j \in \mathcal{J}_N$, J_j is called an inserted job in π if there is a job $J_i \in \mathcal{J}_O$ such that J_j is processed before J_i in π, i.e., $C_j(\pi) < C_i(\pi)$.

Introduction and . . .

The existing . . .

$1|r_j, D_{\max}(\pi^*) \leq \ldots$

Home Page

Title Page

Page 12 of 36

Go Back

Full Screen

Close

Quit

- Write $\mathcal{F}(\sigma) = \{J_j : \ J_j \text{ is an inserted job in } \sigma\}$.

Lemma 2 There is an optimal regular schedule π for the problem $1|r_j, D_{\max}(\pi^*) \leq k|C_{\max}$ such that the jobs in $\mathcal{J}_O \cup \mathcal{F}(\pi)$ are sequenced in ERD order with the jobs in $\mathcal{J}_O$ being sequenced by the natural order of their indices.

Proof For an optimal schedule π for the problem $1|r_j, D_{\max}(\pi^*) \leq k|C_{\max}$, we will lose nothing by rescheduling the jobs in $\mathcal{J}_O \cup \mathcal{F}(\pi)$ in ERD order with the jobs in $\mathcal{J}_O$ being sequenced by the natural order of their indices. Hence, the result holds. □

• The property that the original jobs and the inserted jobs are scheduled in ERD order is called the weak ERD property.

• By Lemma 2, we only need to find an optimal regular schedule with the weak ERD property.

• We first introduce some notations. Let $\mathcal{E}$ be a set of jobs with release dates. Suppose that the jobs in $\mathcal{E}$ have been regularly scheduled by the ERD rule. Under the ERD schedule $ERD(\mathcal{E})$, let S_j and C_j be the starting time and completion time of a job $J_j \in \mathcal{E}$, respectively. Then each job $J_j \in \mathcal{E}$ occupies a time interval $[S_j, C_j)$ in $ERD(\mathcal{E})$.

Recall that $C(\mathcal{E}) = \max\{C_j : J_j \in \mathcal{E}\}$ is the makespan of $ERD(\mathcal{E})$.

• The benefit of the assumption $\mathcal{J}_N = \mathcal{J}_M$ is that, if π is a regular schedule of $\mathcal{J}$, then we have

$$C_{\max}(\pi) = C(\mathcal{J}_O \cup \mathcal{F}(\pi)) + P(\mathcal{J}_N \setminus \mathcal{F}(\pi)),$$

where $P(\mathcal{J}_N \setminus \mathcal{F}(\pi)) = \sum_{J_j \in \mathcal{J}_N \setminus \mathcal{F}(\pi)} p_j$.

Introduction and . . .
The existing . . .
$1|r_j, D_{\max}(\pi^*) \leq \ldots$
Home Page
Title Page
◀◀ ▶▶
◀ ▶
Page 14 of 36
Go Back
Full Screen
Close
Quit

• The reason for the above statement is that, $r_j < r_{n_O}$ for $J_j \in \mathcal{J}_N \setminus \mathcal{F}(\pi)$, and so there are no idle times after job J_{n_O} in the schedule π.

• In the sequel, a subset of $[0, C(\mathcal{E})) \setminus \bigcup_{J_j \in \mathcal{E}} [S_j, C_j)$ is called an idle-time space.

• Especially, for $0 \le t \le C(\mathcal{E})$, we use $I(\mathcal{E}, t)$ to denote the maximal idle-time space after the time moment t, i.e., $I(\mathcal{E}, t) = [t, C(\mathcal{E})) \setminus \bigcup_{J_j \in \mathcal{E}} [S_j, C_j)$.

• We also use $L(\mathcal{E}, t)$ to denote the length of $I(\mathcal{E}, t)$.

• For example, if $\mathcal{E} = \{J_a, J_b, J_c\}$ with $r_a = 1, p_a = 1, r_b = 4, p_b = 2, r_c = 7, p_c = 3$, then, as shown in Figure 1, we have $C(\mathcal{E}) = 10$, $I(\mathcal{E}, 3) = [3, 4) \cup [6, 7)$ and $L(\mathcal{E}, 3) = (4 - 3) + (7 - 6) = 2$.

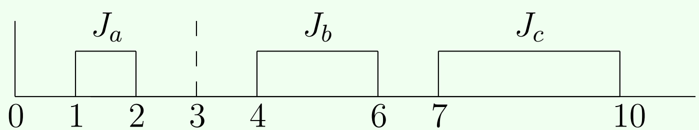

Figure 1.

• From the definition of $I(\mathcal{E}, t)$ and $L(\mathcal{E}, t)$, we can observe that, if $t' > t$ is another time moment such that there are no idle times between t and t' in the schedule $ERD(\mathcal{E})$, then $I(\mathcal{E}, t) = I(\mathcal{E}, t')$ and $L(\mathcal{E}, t) = L(\mathcal{E}, t')$.

Now let us return to the job system $\mathcal{J} = \mathcal{J}_O \cup \mathcal{J}_N$.

• For a job set $\mathcal{E} \subseteq \mathcal{J}$ with $\mathcal{J}_O \subseteq \mathcal{E}$ and a job $J_j \in \mathcal{J}_N \setminus \mathcal{E}$, we write

$$X(\mathcal{E}, J_j) = C(\mathcal{E}) + p_j - C(\mathcal{E} \cup \{J_j\}).$$

• $X(\mathcal{E}, J_j)$ can be interpreted as the length of the idle-time space in $ERD(\mathcal{E})$ which is occupied when job J_j is inserted.

Home Page
Title Page
Go Back
Full Screen
Close
Quit

• Similarly, for a subset $\mathcal{F} \subseteq \mathcal{J} \setminus \mathcal{E}$, we write

$$X(\mathcal{E}, \mathcal{F}) = C(\mathcal{E}) + P(\mathcal{F}) - C(\mathcal{E} \cup \mathcal{F}),$$

where $P(\mathcal{F})$ is the total processing time of the jobs in $\mathcal{F}$.

• Also, $X(\mathcal{E}, \mathcal{F})$ can be interpreted as the length of the idle-time space in $ERD(\mathcal{E})$ which is occupied when the jobs in $\mathcal{F}$ is inserted.

• For a regular schedule σ with the weak ERD property, the makespan $C_{\max}(\pi)$ is calculated by

$$\begin{aligned} C_{\max}(\pi) &= C(\mathcal{J}_O \cup \mathcal{F}(\pi)) + P(\mathcal{J}_N \setminus \mathcal{F}(\pi)) \\ &= C(\mathcal{J}_O) + P(\mathcal{F}(\pi)) - X(\mathcal{J}_O, \mathcal{F}(\pi)) + P(\mathcal{J}_N \setminus \mathcal{F}(\pi)) \\ &= C(\mathcal{J}_O) + P(\mathcal{J}_N) - X(\mathcal{J}_O, \mathcal{F}(\pi)). \end{aligned}$$

• Hence, the target of the problem $1|r_j, D_{\max}(\pi^*) \leq k|C_{\max}$ is equivalent to find a subset $\mathcal{F}$ of $\mathcal{J}_N$ of cardinality at most k such that $X(\mathcal{J}_O, \mathcal{F})$, the length of the idle-time space in the ERD schedule of the jobs in $\mathcal{J}_O$ which is occupied when the jobs in $\mathcal{F}$ is inserted into $\mathcal{J}_O$, is as large as possible.

• Based on this observation, the idea of the algorithm in this paper is that, beginning from $\mathcal{E} := \mathcal{J}_O$ and always choosing the job $J_i \in \mathcal{J}_N \setminus \mathcal{E}$ to be inserted such that $X(\mathcal{E}, J_j)$ is as large as possible.

The following lemma can be observed.

Lemma 3 For a job set $\mathcal{E}$ with $\mathcal{J}_O \subseteq \mathcal{E}$ and a job $J_j \in \mathcal{J}_N \setminus \mathcal{E}$, we have

$$X(\mathcal{E}, J_j) = \min\{p_j, L(\mathcal{E}, r_j)\}.$$

Lemma 4 Suppose that $\mathcal{E}$ and $\mathcal{E}'$ are two sets of jobs such that $\mathcal{J}_O \subseteq \mathcal{E} \subseteq \mathcal{E}'$. Suppose further that $J_j \in \mathcal{J}_N \setminus \mathcal{E}'$. Then

$$X(\mathcal{E}, J_j) \geq X(\mathcal{E}', J_j).$$

Proof Let π be the ERD schedule for the jobs in $\mathcal{E}$ and π' be the ERD schedule for the jobs in $\mathcal{E}'$. Clearly, the length of the idle-time space after r_j in π is greater than or equal to that in π'. Hence, we have $X(\mathcal{E}, J_j) \geq X(\mathcal{E}', J_j)$. □

In the above two lemmas, Lemma 3 states how much idle time will be occupied by an inserted job; whereas, Lemma 4 states that inserting a new job J_j to a set $\mathcal{E}$ occupies at least as much as idle time as inserting it to a superset of $\mathcal{E}$.

The following lemma is critical.

Lemma 5 Suppose that $\mathcal{E}$ is a job set with $\mathcal{J}_O \subseteq \mathcal{E}$ such that there is an optimal regular schedule π of $\mathcal{J} = \mathcal{J}_O \cup \mathcal{J}_N$ with the weak ERD property such that all jobs in $\mathcal{E} \setminus \mathcal{J}_O$ are inserted jobs, i.e., $\mathcal{E} \setminus \mathcal{J}_O \subseteq \mathcal{F}(\pi)$. Suppose further that $J_j \in \mathcal{J}_N \setminus \mathcal{E}$ and

$$X(\mathcal{E}, J_j) = \max\{X(\mathcal{E}, J_i) : J_i \in \mathcal{J}_N \setminus \mathcal{E}\}.$$

If $|\mathcal{E} \setminus \mathcal{J}_O| \leq k - 1$, then there exists an optimal regular schedule with the weak ERD property such that all jobs in $(\mathcal{E} \setminus \mathcal{J}_O) \cup \{J_j\}$ are inserted jobs, i.e., $(\mathcal{E} \setminus \mathcal{J}_O) \cup \{J_j\} \subseteq \mathcal{F}(h)$.

Introduction and . . .
The existing . . .
$1|r_j, D_{\max}(\pi^*) \leq \ldots$
Home Page
Title Page
Page 21 of 36
Go Back
Full Screen
Close
Quit

Proof Let π be an optimal schedule $\mathcal{J} = \mathcal{J}_O \cup \mathcal{J}_N$ with the weak ERD property such that all jobs in $\mathcal{E} \setminus \mathcal{J}_O$ are inserted jobs. If J_j is also an inserted job in π, we have nothing to do. Hence, we suppose that J_j is not an inserted job in π. Let $\mathcal{F} = \mathcal{F}(\pi)$ be the set of all inserted jobs in π. Then $\mathcal{E} \setminus \mathcal{J}_O \subseteq \mathcal{F}$, and

$$C_{\max}(\pi) = C(\mathcal{J}_O \cup \mathcal{F}) + P(\mathcal{J}_N \setminus \mathcal{F}).$$

- If $|\mathcal{F}| < k$, then we define a regular schedule h (with the weak ERD property) from π by adding J_j as an inserted job, i.e., the jobs in $\mathcal{J}_O \cup \mathcal{F} \cup \{J_j\}$ are regularly scheduled in ERD order and the other jobs are regularly scheduled after the jobs in $\mathcal{J}_O \cup \mathcal{F} \cup \{J_j\}$ in any order. Then we have

$$C_{\max}(h) = C(\mathcal{J}_O \cup \mathcal{F} \cup \{J_j\}) + P(\mathcal{J}_N \setminus (\mathcal{F} \cup \{J_j\})).$$

Consequently, we have

$$\begin{aligned} & C_{\max}(\pi) - C_{\max}(h) \\ = \ & C(\mathcal{J}_O \cup \mathcal{F}) + p_j - C(\mathcal{J}_O \cup \mathcal{F} \cup \{J_j\}) \\ = \ & X(\mathcal{J}_O \cup \mathcal{F}, J_j) \\ \geq \ & 0. \end{aligned}$$

It follows that h is also an optimal regular schedule with the weak ERD property such that the jobs in $(\mathcal{E} \setminus \mathcal{J}_O) \cup \{J_j\}$ are inserted jobs, i.e., $(\mathcal{E} \setminus \mathcal{J}_O) \cup \{J_j\} \subseteq \mathcal{F}(h)$, as required.

• In the following, we suppose that $|\mathcal{F}| = k$. Then $\mathcal{F} \setminus \mathcal{E}$ is not empty. We will show that there are a job $J_i \in \mathcal{F} \setminus \mathcal{E}$ and an optimal regular schedule h with the weak ERD property such that $(\mathcal{F} \setminus \{J_i\}) \cup \{J_j\} = \mathcal{F}(h)$. This leads to the conclusion of Lemma 5, since $(\mathcal{E} \setminus \mathcal{J}_O) \cup \{J_j\} \subseteq (\mathcal{F}(\pi) \setminus \{J_i\}) \cup \{J_j\}$.

Let $J_i \in \mathcal{F} \setminus \mathcal{E}$ be such that $C_i(\pi)$ is as large as possible, i.e., J_i is the last inserted job in $\mathcal{F} \setminus \mathcal{E}$. The starting time of J_i in π is $S_i(\pi) = C_i(\pi) - p_i$. By pushing J_j into the set of inserted jobs and pulling J_i out off the set of inserted jobs, we obtain a new schedule σ with the weak ERD property.

Recall that $X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}, J_x)$, for $J_x \notin \mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})$, is the length of the idle time space will be occupied by inserting job J_x to the regular ERD schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$. We have

$$C_{\max}(\pi) = C(\mathcal{J}_O \cup \mathcal{F}) + P(\mathcal{J}_N \setminus \mathcal{F})$$

and

$$C_{\max}(\sigma) = C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}) \cup \{J_j\}) + P(\mathcal{J}_N \setminus \mathcal{F}) - p_j + p_i.$$

Since

$$C(\mathcal{J}_O \cup \mathcal{F}) = C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})) + p_i - X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i)$$

and

$$C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}) \cup \{J_j\}) = C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})) + p_j - X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j),$$

we have

$$\begin{aligned} & C_{\max}(\pi) - C_{\max}(\sigma) \\ = \; & C(\mathcal{J}_O \cup \mathcal{F}) + p_j - C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}) \cup \{J_j\}) - p_i \\ = \; & X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j) - X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i). \end{aligned}$$

In order to show that σ is an optimal regular schedule with the weak ERD property such that the jobs in $(\mathcal{F} \setminus \{J_i\}) \cup \{J_j\}$ are inserted jobs in σ, we only need to prove that

$$(*): \qquad X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j) \geq X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i).$$

If

$$C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})) > C(\mathcal{E}),$$

we claim that there are no idle times after r_i in the schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$, and thus $X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})), J_i) = 0$.

Suppose, to the contrary, that there are idle times after r_i in the schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$. Since $C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})) > C(\mathcal{E})$, $\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}) \neq \mathcal{E}$. Hence, $|\mathcal{F} \setminus \{J_i\}| > |\mathcal{E} \setminus \mathcal{J}_O|$.

Let J_x be the second last inserted job in $\mathcal{F} \setminus \mathcal{E}$, i.e., $J_x \in \mathcal{F} \setminus \mathcal{E}$ with $x \neq i$ such that $C_i(\pi)$ is as large as possible. By the weak ERD property of π, we have $r_x \leq r_i$. This means that there are idle times after r_x in the schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$. Since there are no idle times between r_x and S_x (the starting time of J_x) in the schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$, there are idle times after job J_x in the schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$. Let t be the earliest time moment such that there are no idle times after t in the schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$. Then J_x is completed before t in the schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$. Write

$$\mathcal{J}^t = \{J_y \in \mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}) :\ C_y \geq t \text{ in } ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))\}.$$

Then

$$\min\{r_y : J_y \in \mathcal{J}^t\} = t$$

and $\mathcal{J}^t \subseteq \mathcal{E}$. Hence, we have

$$C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})) = C(\mathcal{J}^t) \leq C(\mathcal{E}),$$

a contradiction.

We conclude the statement that there are no idle times after r_i in the schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$. Consequently, $X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})), J_i) = 0$. It follows that

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j) \geq X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i),$$

i.e., $(*)$ holds in the assumption that $C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})) > C(\mathcal{E})$, as required.

In the following, we assume that

$$C(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\})) = C(\mathcal{E}).$$

Introduction and . . .
The existing . . .
$1|r_j, D_{\max}(\pi^*) \leq \ldots$
Home Page
Title Page
Page 28 of 36
Go Back
Full Screen
Close
Quit

We distinguish the following two cases.

Case 1 $S_i(\pi) \geq r_j$.

In this case, since there are no idle times between r_i and $S_i(\pi)$, we have $L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), r_i) = L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), S_i(\pi))$. Furthermore, we can observe that

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i) = \min\{X(\mathcal{E}, J_i), L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), r_i)\}$$

and

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j) = \min\{X(\mathcal{E}, J_j), L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), r_j)\}.$$

Since

$$X(\mathcal{E}, J_j) \geq X(\mathcal{E}, J_i)$$

and

$$L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), r_j) \geq L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), S_i(\pi)),$$

we have

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j) \geq X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i),$$

i.e., $(*)$ holds.

Case 2 $S_i(\pi) < r_j$.

In this case, we consider two subcases.

Case 2.1 There are idle times between r_i and r_j in the schedule $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$.

In this subcase, the two schedules $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$ and $ERD(\mathcal{E})$ are the same after the time moment r_j, since J_i is the last job inserted in $\mathcal{F} \setminus \mathcal{E}$ in π. Hence we have

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j) = X(\mathcal{E}, J_j).$$

By Lemma 4, we further have

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i) \leq X(\mathcal{E}, J_i).$$

From the fact

$$X(\mathcal{E}, J_j) \geq X(\mathcal{E}, J_i),$$

we deduce that

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j) \geq X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i),$$

i.e., $(*)$ holds.

Introduction and . . .
The existing . . .
$1|r_j, D_{\max}(\pi^*) \leq \ldots$
Home Page
Title Page
Page 30 of 36
Go Back
Full Screen
Close
Quit

Case 2.2 There are no idle times between r_i and r_j in $ERD(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}))$.

In this subcase, since there are no idle time between r_i and r_j, we have $L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), r_i) = L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), r_j)$. We can further observe that

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i) = \min\{X(\mathcal{E}, J_i), L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), r_i)\}$$

and

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j) = \min\{X(\mathcal{E}, J_j), L(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), r_j)\}.$$

From the fact

$$X(\mathcal{E}, J_j) \geq X(\mathcal{E}, J_i),$$

we deduce that

$$X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_j) \geq X(\mathcal{J}_O \cup (\mathcal{F} \setminus \{J_i\}), J_i),$$

i.e., $(*)$ holds. $\square$

Algorithm for $1|r_j, D_{\max}(\pi^*) \leq k|C_{\max}$:

Step 1 Set $\mathcal{E} := \mathcal{J}_O$, $i := 0$.

Step 2 If $i = \min\{k, n_N\}$, then go to Step 5.

Step 3 Choose $J_j \in \mathcal{J}_N \setminus \mathcal{E}$ such that $X(\mathcal{E}, J_j)$ is as large as possible.

Step 4 Set $\mathcal{E} := \mathcal{E} \cup \{J_j\}$, $i := i + 1$. Return to Step 2.

Step 5 Sequence the jobs in $\mathcal{E}$ and $\mathcal{J}_N \setminus \mathcal{E}$ in ERD orders π_1 and π_2, respectively.

Step 6 The final regular schedule with the weak ERD property is given by (π_1, π_2). □

Home Page
Title Page
◀◀ ▶▶
◀ ▶
Page 32 of 36
Go Back
Full Screen
Close
Quit

The above algorithm begins from $\mathcal{E} := \mathcal{J}_O$ and $i := 0$. This guarantees that $\mathcal{E} \setminus \mathcal{J}_O \subseteq \mathcal{F}(\pi)$ for some optimal schedule π with the weak ERD property. In each iteration, the condition $\mathcal{E} \setminus \mathcal{J}_O| = i$ is kept.

If $i = \min\{k, n_N\}$, this just means that $\mathcal{E} \setminus \mathcal{J}_O = \mathcal{F}(\pi)$ for some optimal schedule π with the weak ERD property. By the weak ERD property, we sequence the jobs in $\mathcal{E}$ and $\mathcal{J}_N \setminus \mathcal{E}$ in ERD order π_1 and π_2, respectively, and the final optimal schedule is given by the sequence (π_1, π_2). Of course, π_2 can be chosen as an arbitrary order of the jobs in $\mathcal{J}_N \setminus \mathcal{E}$, since the maximum release date of jobs is r_{n_O}.

Introduction and . . .
The existing . . .
$1|r_j, D_{\max}(\pi^*) \leq \ldots$
Home Page
Title Page
Page 33 of 36
Go Back
Full Screen
Close
Quit

Otherwise, $\mathcal{J}_N \setminus \mathcal{E}$ is not empty, and we choose $J_j \in \mathcal{J}_N \setminus \mathcal{E}$ such that $X(\mathcal{E}, J_j)$ is as large as possible. By Lemma 5, there is an optimal schedule h with the weak ERD property such that $(\mathcal{E} \setminus \mathcal{J}_O) \cup \{J_j\} \subseteq \mathcal{F}(h)$. We set $\mathcal{E} := \mathcal{E} \cup \{J_j\}$ and $i := i + 1$, and enter the next iteration. Hence, the algorithm is correct.

The correctness of the above algorithm is guaranteed by Lemma 5. The running time of Step 2 in the above algorithm is up bounded by $O(n_N(n_O + n_N))$. Hence, the computational complexity of the above algorithm is $O(n_N^2(n_O + n_N))$.

Thank You!