List Edge and List Total Colorings of Some Planar Graphs

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1. Introduction

- Let $G = (V, E)$ be a graph with vertex set $V(G) = V$ and edge set $E(G) = E$. If $v \in V$, we write $d(v) = d_G(v)$ for the degree of v in G, and $\Delta(G)$ and $\delta(G)$ for the maximum and minimum degree in G.

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- The mapping L is said to be a total assignment for the graph G if it assigns a list $L(x)$ of possible colors to each element $x \in V \cup E$. If G has a total coloring ϕ such that $\phi(x) \in L(x)$ for all $x \in V \cup E$, and no two adjacent or incident elements receive the same color, then we say that G is total-L-colorable. Let $f : V \cup E \longrightarrow \mathbb{N}$ be a function into the positive integers. We say that G is total-f-choosable if it is total-L-colorable for every total assignment L satisfying $|L(x)| = f(x)$ for all elements $x \in V \cup E$. The list total chromatic number $\chi_l''(G)$ of G is the smallest integer k such that G is totally-f-choosable when $f(x) = k$ for each $x \in V \cup E$.

- The list chromatic number $\chi_l(G)$ of G and the list edge chromatic number (or list chromatic index) $\chi'_l(G)$ of G are defined similarly in terms of coloring vertices alone, or edges alone, respectively; and so are the concepts of vertex-f-choosability and edge-f-choosability. The ordinary vertex, edge and total chromatic number of G are denoted by $\chi(G)$, $\chi'(G)$ and $\chi''(G)$, respectively.

- Part (a) of the following conjecture was formulated independently by Vizing, by Gupta, by Albertson and Collins, and by Bollobás and Harris, and it is well known as the List Coloring Conjecture and part (b) was formulated by Borodin, Kostochka and Woodall.

Conjecture 1.1. If G is a multigraph, then

(a) $\chi'_l(G) = \chi'(G)$, (b) $\chi''_l(G) = \chi''(G)$.

- Vizing's Theorem show that if G is a graph with maximum degree Δ, then either $\chi'(G) = \Delta$ or $\chi'(G) = \Delta + 1$. So Vizing proposed weaker conjecture as following.

Conjecture 1.2**.** Every graph G is edge-$(\Delta(G) + 1)$-choosable.

- In section 2, we give some results about conjectures. In section 3, we give some recent results about conjectures which proved by Hou, Liu and Wu. In section 4, we proposed some new problems.

2. Some Known Results About Conjectures

2.1. Some Known Results About Conjecture 1.1

- List Coloring Conjecture has been proved for a few special cases, such as bipartite multigraphs [1995, Galvin], complete graphs of odd order [1997, Hǻggkvist and Janssen], multicircuits [1999, Woodall], line-perfect multigraphs [1999, Woodall], graphs with $\Delta \geq 12$ which can be embedded in a surface of nonnegative characteristic [1997, Borodin, Kostochka and Woodall] and outerplanar graphs [2001, Wang and Lih].

- Part (b) of Conjecture 1.1 has been proved for outerplanar graphs [2001, Wang and Lih], graphs with $\Delta \geq 12$ which can be embedded in a surface of nonnegative characteristic [1997, Borodin, Kostochka and Woodall].
- For planar graphs, Borodin, Kostochka and Woodall also obtained several related results of Conjecture 1.1 by adding grith restrictions.

2.2. Some Known Results About Conjecture 1.2

- An earlier result of Harris showed that $\chi_l'(G) \leq 2\Delta(G) - 2$ if G is a graph with $\Delta(G) \geq 3$ [1984, Harris]. This implies Conjecture 1.2 for the case $\Delta(G) = 3$. Juvan et al. settled the case of $\Delta(G) = 4$ [1999, Juvan, Mohar and Skrelkovski].
- Conjecture 1.2 has also been confirmed for other special cases such as complete graphs [1997, Hăggkvist and Janssen], graphs with grith at least $8\Delta(G)(\ln\Delta(G) + 1.1)$ [1992, Kostochka] and planar graphs with $\Delta(G) \geq 9$ [1990, Borodin].

- Wang and Lih proved if G is a planar graph with $\Delta(G) \neq 5$ and without two 3-cycles sharing a common vertex, then G is edge-$(\Delta(G)+1)$-choosable. Suppose that G is a planar graph without k-cycles for some fixed integer $3 \leq k \leq 6$. Then it was shown that Conjecture 1.2 holds if G satisfies one of following conditions: (i) either $k = 3$ or $k = 4$ and $\Delta(G) \neq 5$ [2004, Zhang and Baoyindureng]; (ii) $k = 5$ [2002, Wang and Lih]; (iii) $k = 6$ and $\Delta(G) \neq 5$ [2001, Wang and Lih]; (iv) $k = 4$ and $\Delta(G) = 5$, $k = 6$ and $\Delta(G) = 5$ [2005, Wang and Lih].

3. Recent Results About Conjectures

- Note that the added grith requirement prohibits the appearance of triangles. The forbidden cycle or the grith restriction plays an important role in considering list coloring planar graphs. Next we give some results on list edge and list total coloring of planar graphs without certain cycles.

- **Theorem** 3.1 **[2006, Hou, Liu, Wu].** Let G be a planar graph with maximum degree Δ such that G has no cycle of length from 4 to k, where $k \geq 4$. If

  (1) $\Delta \geq 7$ and $k \geq 4$, or

  (2) $\Delta \geq 6$ and $k \geq 5$, or

  (3) $\Delta \geq 5$ and $k \geq 8$,

  then $\chi'_l(G) = \Delta$ and $\chi''_l(G) = \Delta + 1$. Furthermore, if

  (4) $\Delta \geq 4$ and $k \geq 14$,

  then $\chi'_l(G) = \Delta$.

- Remark. (a) In above theorem, we only consider planar graphs. In fact, it is true for graph embedded in a surface of nonnegative characteristic. (b) If we just consider edge list coloring of G, then $k = 7$ is enough in (3). (c) In (4), $\chi_l''(G) = \Delta + 1$ is also true.

- **Theorem** 3.2 **[2006， Hou, Liu, Wu].** Let G be a planar graph with maximum degree Δ. If G is free of 5-cycles, then $\chi'_l(G) \leq \max\{8, \Delta\}$ and $\chi''_l(G) \leq \max\{9, \Delta + 1\}$.
- **Theorem** 3.3 **[2006, Hou, Liu, Wu].** Let G be a planar graph with maximum degree Δ. If G is free of 6-cycles, then $\chi''(G) \leq \max\{8, \Delta + 1\}$.

- **Theorem** 3.4 **[2005, Hou, Liu, Cai]. Every plane graph** G **with** $\Delta(G) \geq 6$ **and without any adjacent** 3**-cycles is edge-**$(\Delta(G)+1)$**-choosable.**

- **Theorem** 3.5 **[2005, Hou, Liu, Cai].** Every plane graph G with $\Delta(G) \geq 7$ and without 7-cycles is edge-$(\Delta(G)+1)$-choosable.

4. Problems for Further Research

- Sanders and Zhao proved that every planar graph with $\Delta \leq 7$ has total 9-coloring which implies that planar graph with $\Delta \geq 7$ has a total $(\Delta + 2)$-coloring.

Problem 4.1. Let G be a C_5-free planar graph with maximum degree $\Delta \geq 7$. Is it true $\chi''(G) = \Delta + 1$. Furthermore, is it true that $\chi'_l(G) = \Delta$ and $\chi''_l(G) = \Delta + 1$.

Problem 4.2. Let G be a C_6-free planar graph with maximum degree $\Delta \geq 7$. Is it true that $\chi'_l(G) = \Delta$ and $\chi''_l(G) = \Delta + 1$.

- **Problem 4.3.** If G is a planar graph without intersecting triangles with maximum degree Δ, is G edge-Δ-choosable if $\Delta \geq 7$.
- **Problem 4.4.** If G is a planar graph without adjacent triangles, is G edge-6-choosable if $\Delta(G) = 5$.

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- G.Liu, J. Hou, J. Cai, Some Results about f-Critical Graphs, submitted to Networks.
- J. Hou, G. Liu, J. Cai, List Edge and List Total Colorings of Planar Graphs without 4-Cycles, submitted to Theoretical Computer Science.
- J. Hou, G. Liu, J. Cai, Edge-Choosability of Some Plane Graphs with Large Maximum Degree, submitted to Discrete Math.

Thank you!