[S,T]-GRAPH

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1 Introduction of $[s,t]$-$Graph$

- 1. Definition

  ⋆ $[s,t]$-$graph$: A graph G is called $[s,t]$-$graph$, if there are at least t edges in every included subgraphs on s vertices.

  Wang Jianglu and Liu Chunfang gave the definition of the new graph family in 2005.By the definition, we can easily obtain several main propositions about $[s,t]$-$graph$:

  ⋆ (1) $[s,t]$-$graph$ is $[s-1,t]$-$graph$;

  ⋆ (2) $[s,t]$-$graph$ is $[s,t+1]$-$graph$;

  ⋆ (3) $[s,t]$-$graph$ is $[s+1,t+1]$-$graph$;

  ⋆ (4) If H is a subgraph of a $[s,t]$-$graph$ with $|H| \geq s$, H is $[s,t]$-$graph$.

- 2. Application

  ⋆ (1) In graph theory, when we study $Hamilton$ problem, we often try to add the connectivity to assure the being of path(cycle). However, this may not assure the edges exist uniformly. One better proposition of $[s,t]$-$graph$ is that its edges exist more uniformly.

  ⋆ (2) It can be used in traffic network, communications, the configuration of computer networks, etc. For example, if there are at least t paths among s cities, how can we look for a better way from one city to another; and if one of the t paths is broken, is there one path can substitute for it?

2 Path(Cycle) in $[s,t]$-$Graph$

- Previous Results

◇ **Theorem2.1(Liu and Wang, 2005)** Suppose G is a $[4,2]$-$graph$, then
(a) G is connected iff $G \cong K_{1,3}$ or G has a $Hamilton$ path.
(b) G is a 2-connected iff $G \cong K_{2,3}$ or $G \cong K_{1,1,3}$ or G has a $Hamilton$ cycle.

◇ **Theorem2.2(Liu and Wang, 2005)** Suppose G is a 2-connected $[5,2]$-$graph$, then $G \cong K_{2,4}$ or G has a $Hamilton$ path. Moreover, the assumption "2-connected" is best possible.

◇ **Theorem2.3(Liu, 2005)** Suppose G is a k-connected $[k+2,k]$-$graph$, then $G \cong \overline{K_{k+1}} \vee G_k$ or G has a $Hamilton$ cycle.

◇ **Theorem2.4(Lin and kong, 2006)** Suppose G is a 3-connected $[5,3]$-$graph$, then $G \cong \overline{K_4} \vee C_3$ or G has a $Hamilton$ cycle.

- My Work 1

◇ **Theorem2.5(Li, Qu and Wang, 2006)** (1) If G is a connected [5,3]-$graph$ and $|G| \geq 6$, then the longest path of G is at least $n-2$. Moreover, $n-2$ is best possible (See Fig.2.1 and Fig.2.2).

(2) If G is a connected [5,3]-$graph$ and $|G| \geq 6$, then the longest cycle of G is indefinite.

⋆ Example

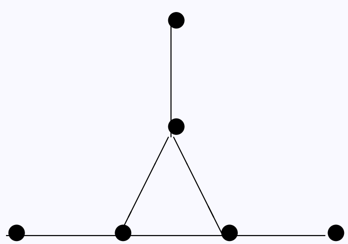

Fig.2.1

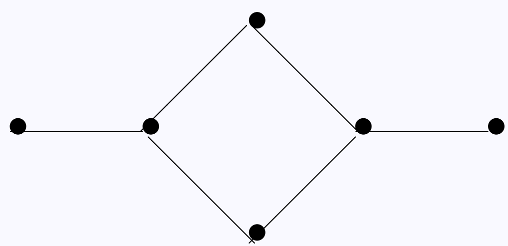

Fig.2.2

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- My Work 2

  ◇ **Theorem2.6(Li and Wang, 2006)** Suppose G is k-connected $[k+2, 1]-graph$， then G has $Hamilton$ path.

  Moreover,the $k+2$ is best possible.If we changed $k+2$ to $k+3$,the 1 can add to $2, 3, 4$,even bigger.$K_{2,4}$ can show this.

  ◇ **Theorem2.7(Li and Wang, 2006)** Suppose G is k-connected $[k+1, 1]-graph$， then G has $Hamilton$ cycle.

  Moreover,the $k+1$ is best possible.If we changed $k+1$ to $k+2$,the 1 can add to $2, 3, 4$,even bigger.$K_{2,3}$ can show this.

- My Work 3

◇ **Theorem2.8(Li and Wang, 2006)** If G is a 2-connected [5,3]-*graph* with $|G| \geq 8$ and $\delta(G) \geq 3$, then G has a $Hamilton$ cycle.

$K_{2,3}$, Fig.2.3 and Fig.2.3 show that the assumptions "$\delta(G) \geq 3$","2-connected" and "$|G| \geq 8$" are best possible,respectively. Fig.2.4 is a 2-connected [5,2]-graph which doesn't contain a $Hamilton$ cycle.

⋆ Example

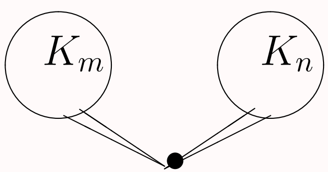

Fig.2.3 $(m, n \geq 1)$

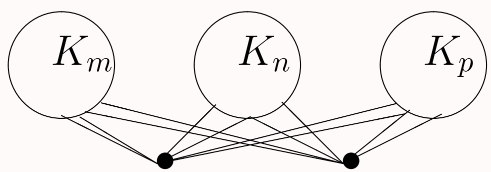

Fig.2.4 $(m, n, p \geq 1)$

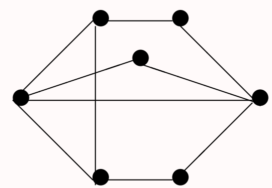

Fig.2.5

3 Path(Cycle) Extendence in $[s, t]$-*graph*

- Previous Results

  ◇ **Theorem3.1(Wang, 1997)** Every connected, locally 2-connected, claw-free graph is path extendable.

  ◇ **Theorem3.2(You, 2003)** Every connected, locally 3-connected, $(K_{1,4;2})$ graph is path extendable.

  ◇ **Theorem3.3(Hendry, 1990)** If G is a connected, locally connected claw-free graph with $|G| \geq 3$, then G is fully cycle extendable.

  ◇ **Theorem3.4(Ryj*áč*ek, 1994)** If G is a connected, locally connected almost claw-free graph with $|G| \geq 3$, then $K_{1,4} \subseteq G$ or G is fully cycle extendable.

- My Work 3

◇ **Theorem3.5(Li and Wang, 2006)** If G is a connected, locally 2-connected $[4,2]$-$graph$, then $K_{1,1,1,3} \subseteq G$ or G is path extendable.

$K_{1,1,3}$ shows the assumption "locally 2-connected" is best possible; Fig.3.1 is an example which satisfies the condition of Theorem 3.5, but not satisfies those of Theorem 3.1.

⋆ Example

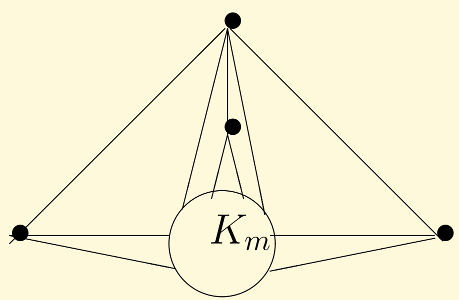

Fig.3.1$(m \geq 2)$

- My Work 4

◇ **Theorem3.6(Li and Wang, 2006)** If G is a connected, locally connected $[5,3]-graph$ with $\delta(G) \geq 3$, then $G \cong F$ or G is fully cycle extendable (See F in Fig.3.2).

$K_{1,1,3}$ shows the assumption "$\delta(G) \geq 3$" is best possible;Fig.3.3 is an example which satisfies the condition of Theorem 3.6, but not satisfies those of Theorem 3.4.

⋆ Example

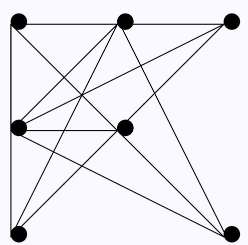

Fig. 3.2

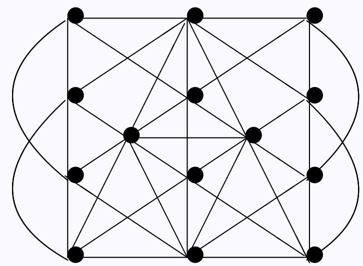

Fig. 3.3

4 Vertex-Disjoint Cycles In $[4,2]$-*graph*

- Previous Results

◇ **Theorem4.1(Corrádi and Hajnal, 1963)** Suppose $|G| = n \geq 3k$ and $\delta(G) \geq 2k$. Then G contains $k\ vertex-disjoint$ cycles.

◇ **Theorem4.2(Brandt, 1997)** Suppose $|G| = n \geq 4k$ and $\sigma_2(G) \geq n$. Then G can be partitioned into $k\ vertex-disjoint$ cycles, there exist $k\ vertex-disjoint$ cycles $H_1, H_2, \cdots, H_k$ $such\ that$ $V(G) = \cup_{i=1}^{k} V(H_i)$.

To prove the theorem, they used the following result:

◇ **Theorem4.3(Justesen, 1996)** Suppose $|G| = n \geq 3k$ and $\sigma_2(G) \geq 4k$. Then G contains $k\ vertex-disjoint\ cycles$.

◇ **Theorem4.4(Enomoto, 1998)** Suppose $|G| = n \geq 3k$ and $\sigma_2(G) \geq 4k-1$. Then G contains $k\ vertex-disjoint$ cycles, and the assumption $\sigma_2(G) \geq 4k-1$ is weakest possible.

- My Work 4

◇ **Theorem4.5(Li min, 2006)** Let G be a $[4,2]$-graph, suppose $|G| = n \geq 3k$ and $\sigma_2(G) \geq 3k+1$. Then G contains k $vertex-disjoint$ cycles. Fig.4.1 does not contain k $vertex-disjoint$ cycles. Since the graph in Fig.4.2 does not contain k $vertex-disjoint$ cycles, then $\sigma_2(G) \geq 3k$.

⋆ Example

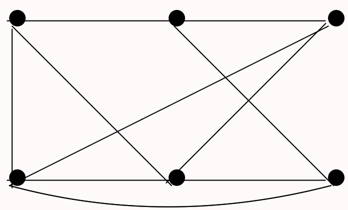

Fig. 4.1

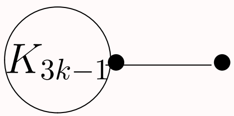

Fig. 4.2

5 Note

There are few results about $[s, t]$-$graph$, since it is a new graph family. In my opinion, more and better results about $[s, t]$-$graph$ will be obtained through our joint and hard work. If we can know the extremal graph of the $[s, t]$-$graph$ or more results about $[s, t]$-$graph$ when s, t are normal, it will be much better!

Thank You !