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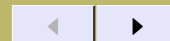
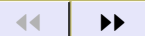
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On the Spectra of Bicyclic Graphs

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- The sharp lower bound of the least eigenvalue of a bicyclic graph
- On the spectral radius of bicyclic graphs with n vertices and diameter d

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1 Preliminaries

- Let $G = (V(G), E(G))$ be a simple graph with n vertices, and let $A(G)$ be a $(0, 1)$ -adjacency matrix of G with $a_{ij} = 1$ for $v_i \sim v_j$ and $a_{ij} = 0$ otherwise.
- The characteristic polynomial of G is just $\det(\lambda I - A(G))$, and is denoted by $\phi(G; \lambda)$.

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- The eigenvalues of $A(G)$ is called that of G . In particular, the largest eigenvalue of $A(G)$ is called the spectral radius of G and denoted by $\rho(G)$, the least eigenvalue of $A(G)$ is denoted by $\lambda(G)$.
- The diameter of a connected graph is the maximum distance between pairs of its vertices.
- Bicyclic graphs are connected graphs in which the number of edges equals the number of vertices plus one.

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2 | The sharp lower bound of the least eigenvalue of a bicyclic graph

- As G ranges over the collection of all simple graphs with n vertices, it is often required to know how small $\lambda(G)$ gets.

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- Brigham and Dutton (JCT(B) 37 (1984)) proved

$$\lambda(G) \geq \max \left\{ -\sqrt{\frac{2mn_-}{1+n_-}}, -\sqrt{\frac{2mn_+}{1+n_+}} \right\},$$

where n_+, n_- are the numbers of positive eigenvalues and negative eigenvalues of G , respectively.

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- Constantine (LAA 65(1985)) showed that

$$\lambda(G) \geq -n/2 \quad \text{if } n \text{ is even}$$

or

$$\lambda(G) \geq -\sqrt{n^2 - 1}/2 \quad \text{if } n \text{ is odd.}$$

- The same result was obtained by different means by Y. Hong (Acta Math. Appl. Sinica 4(1998)) and P. L. Powers (LAA 101(1988)).

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- For some special families of graphs, the above problem has also been studied by several authors.
- Let G be a simple graph with $n \geq 3$ vertices, orientable genus g and non-orientable genus h . Hong and Shu (LAA 296 (1999)(1-3)) defined the Euler characteristic $\mathcal{X}(G)$ of a graph G by $\mathcal{X}(G) = \max\{2 - 2g, 2 - h\}$, and proved that $\lambda(G) \geq -\sqrt{2(n - \mathcal{X}(G))}$.

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- In particular, if G is a planar graph, they obtained $\lambda(G) \geq -\sqrt{2n-4}$ with equality if and only if $G = K_{2,n-2}$. For series-parallel graphs, they obtained a similar result.
- Xu, Xu and Wang gave a sharp lower bound on the least eigenvalue of a unicyclic graph with n vertices and characterized the graph attained the lower bound. (J. of Ningbo University(NSEE). 16(3) (2003))

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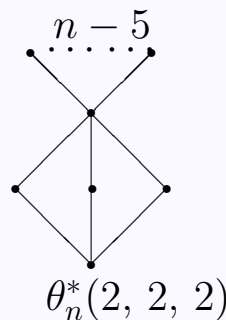
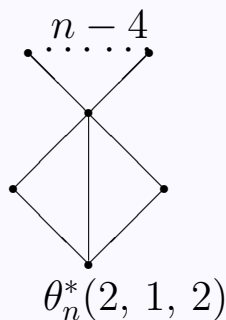
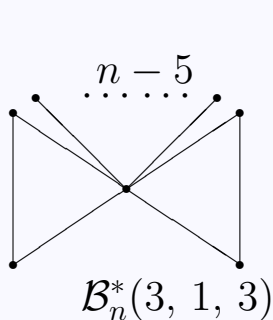
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• **Theorem 1 (S. G. Guo).** *Let $n > 8$, G be a bicyclic graph on n vertices and $G \neq B_n^*(3, 1, 3), \theta_n^*(2, 1, 2)$. Then*

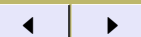
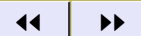
$$\lambda(G) \geq -\sqrt{\frac{n+1+\sqrt{(n-5)^2+36}}{2}}$$

with the equality if and only if $G = \theta_n^(2, 2, 2)$. (to appear in Ars Comb.)*



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• **Theorem 2 (S. G. Guo).** Let G be a bicyclic graph on n vertices.

(1) If $8 \leq n \leq 27$ and $G \neq \theta_n^*(2, 2, 2)$, then $\lambda(G) > \lambda(\theta_n^*(2, 2, 2))$.

(2) If $28 \leq n \leq 29$ and $G \neq \theta_n^*(2, 2, 2)$, $\theta_n^*(2, 1, 2)$, then

$$\lambda(G) > \lambda(\theta_n^*(2, 2, 2)) > \lambda(\theta_n^*(2, 1, 2)).$$

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- (3) If $n \geq 30$ and $G \neq \theta_n^*(2, 2, 2)$,
 $B_n^*(3, 1, 3)$, $\theta_n^*(2, 1, 2)$, then

$$\begin{aligned} \lambda(G) &> \lambda(\theta_n^*(2, 2, 2)) > \lambda(B_n^*(3, 1, 3)) \\ &> \lambda(\theta_n^*(2, 1, 2)). \end{aligned}$$

(to appear in *Ars Comb.*)

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3 | On the spectral radius of bicyclic graphs with n vertices and diameter d

- Recently, the problem of finding all graphs with maximal or minimal spectral radius among a given class of graphs has been studied extensively.
- Simić determined the bicyclic graphs on prescribed number of vertices with the largest eigenvalue minimal. (Publ. Inst. Math. (Beograd) (N. S.) 46(60)(1989))

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- Chang, Tian and Yu determined graphs with the largest spectral radius among all the bicyclic graphs on n vertices with perfect matching. (Discrete. Math. 283(2004))
- Yu and Tian determined the graph with the largest spectral radius among all the bicyclic graphs on n vertices with a maximum matching of cardinality m . (MATCH Commun. Math. Comput. Chem. 52(2004))

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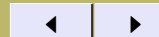
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- S. G. Guo, Petrović and Gutman determined the graph with the largest spectral radius among all the bicyclic graphs with n vertices and k pendant vertices. (LAA 408(2005); Bull. Acad. Serbe Sci. Arts (Cl. Sci. Math. Natur.) 22(IV)(2005))
- S. G. Guo, Xu and Chen determined the graphs with the largest and the second largest spectral radius among all trees with n vertices and diameter d . (Advance in Math. (China) 34(6)(2005))



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- J. M. Guo and Shao characterized the first $\lfloor \frac{d}{2} \rfloor + 1$ trees with n vertices and diameter d ordered according to their spectral radii. (LAA 413(1)(2006))
- S. G. Guo determined the graph with the largest spectral radius among all unicyclic graphs with n vertices and diameter d . (Australas. J. Combin. submitted, 2005)

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• We denote by $\mathcal{B}_{n,d}$ the set of all bicyclic graphs with n vertices and diameter d .

• $P_{d+1}^+(i), P_4^\theta(3)$

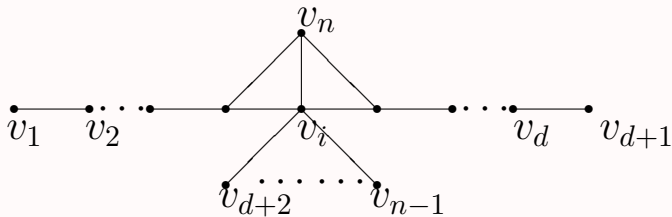


Fig. 7 $P_{d+1}^+(i)$

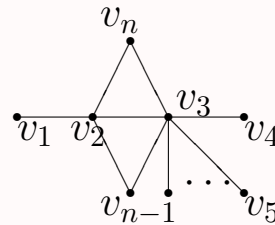


Fig. 6 $P_4^\theta(3)$

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• **Theorem 3 (S. G. Guo).** Let $n \geq d + 5$ and $G \in \mathcal{B}_{n,d}$. If $d \geq 4$, then

$$\rho(G) \leq \rho(P_{d+1}^+(\lfloor \frac{d+2}{2} \rfloor))$$

with equality if and only if $G = P_{d+1}^+(\lfloor \frac{d+2}{2} \rfloor)$. If $d = 3$, then

$$\rho(G) \leq \rho(P_4^\theta(3))$$

with equality if and only if $G = P_4^\theta(3)$.
(to appear in LAA)

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谢谢大家!



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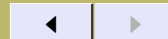
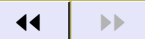
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