# A construction of an Error-correcting Pooling Design

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# 1 Introduction

- For $k \in [n]$, $\left(\binom{n}{k}\right)$ denotes the family of $k-$sets of $[n]$, and $\left[\begin{bmatrix}n\\d\end{bmatrix}\right]$ denotes the family of subsets of $[n]$ with cardinality at most $k$.
- The *boolean sum* of two $n-$ vector $\mathbf{x}$ and $\mathbf{y}$ is defined coordinate-wise using the rule $\mathbf{x_i} \vee \mathbf{y_i} = 0$ if and only if both $\mathbf{x_i}$ and $\mathbf{y_i}$ are zero; $\mathbf{x_i} \vee \mathbf{y_i} = 1$ otherwise.
- The complement $\mu^c$ of a matrix $\mu$ on $[n]$ is the matrix that results when one interchanges the $0s$ and $1s$ in $\mu$.
- Given a matrix $\mu$ on $[n]$, let $B_d(\mu)$ denote a $0, 1$ matrix whose columns consist exactly of all the boolean sums $\bigvee_{j \in S} \mathbf{c}_j(\mu)$ where $S \in \left[\begin{bmatrix}n\\d\end{bmatrix}\right]$.
- For $n-$vectors $\mathbf{x}$ and $\mathbf{y}$, let $H(\mathbf{x}, \mathbf{y})$ denote the number of corresponding components of $\mathbf{x}$ and $\mathbf{y}$ that are different. $H(\mathbf{x}, \mathbf{y})$ is known as the *Hamming distance*[1]. For a matrix $\mu$ on $[n]$, let $H(\mu)$[1] denote the minimum Hamming distance between any pair of columns of $\mu$.

- Let $(\binom{n}{j})$ denote the family of $j-$subsets of $[n]$. For $d < k < n$, we define the $\binom{n}{d} \times \binom{n}{k}$ $\{0,1\}$ matrix $\delta(n,d,k)$([3]) by letting the rows and the columns be, respectively, represented by the members of $(\binom{n}{d})$ and $(\binom{n}{k})$ in the following way: For a given $D \in (\binom{n}{d})$ and $K \in (\binom{n}{k})$, the matrix $\delta(n,d,k)$ has a $1$ in its $(D,K)$th entry if and only if $D \subset K$.

- A $t \times n$ matrix $\mu$ is said to be $d-$disjunct if the union of any $d$ columns does not contain another column. A $t \times n$ $\mu$ is said to be $(d,e)-$disjunct if for any $d+1$ columns $C_0, C_1, \cdots, C_d$ of $\mu$ there are at least $e+1$ elements in $C_0 - \bigcup_{i=0}^{d} C_i$.

- For $1 \leq d < k < n$, reference [1] defined the matrix $\delta^*(n,d,k)$ as that by augmenting the matrix $\delta(n,d,k)$ with $\delta^c(n,1,k)$.

- If $k-d \geq 3$, then $H(B_d(\delta^*(n,d,k))) \geq 4$(see reference [1]).

## 2 Main Results

- Definition: For $1 \leq d < k < n$, we define the matrix $\delta^{**}(n,d,k)$ as that by augmenting the matrix $\delta(n,d,k)$ with $\delta^{c}(n,2,k)$.

- Theorem 1. If $k-d \geq 3$, then $H(B_d(\delta^{**}(n,d,k))) \geq 4$.

- Remark: The condition $k-d \geq 3$ in Theorem 1 can not be abbreviated for the general $k$. For example: The matrix $\delta^{(}7,3,5)$ has column sets $C_1 = \{1,2,3,4,6\}$, $C_2 = \{1,2,3,4,7\}$ and $C_3 = \{1,2,3,5,6\}$. Consider $\{C_1, C_2, C_3\}$ and $\{C_2, C_3\}$. It is obvious that only the elements in rows $\{1,4,5\}$, $\{2,4,5\}$ and $\{3,4,5\}$ are different. So $H(B_d(\delta^{**}(7,3,5))) = 3$.

- Theorem 2. If $\delta^{**}(n,d,k)$ is the matrix by augmenting the matrix $\delta(n,d,k)$ with $\delta^{c}(n,\alpha,k)$, $1 \leq \alpha \leq m+1 (\alpha \in \mathbb{Z})$ and $k-d \geq m (m \geq 3)$, then we have $H(B_d(\delta^{**}(n,d,k))) \geq 4$.


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# 3 References


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