Error-Correcting Nonadptive Pooling Designs Associated with Finite Geometries and Association Schemes

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Outline:

1. nonadaptive *pooling designs* (*group testings*)

2. from *d-disjunct* matrices to *generalized cover free families*

3. pooling spaces and some families of examples

4. pooling designs associated with distance regular graphs

5. Pooling designs on complexes

6. error-correcting and decoding

1. *nonadaptive pooling designs* (*group testings*)

group testing:

to identify the set of (rare) positives in a large population of items by performing

1. a series of {0, 1} tests on subsets (pools) of the population;
2. a test result is 1 (positive) if *a positive* is included in the test pool; and 0 (negative) otherwise.

non-adaptive group testing algorithm:

all test - pools are specified without knowing the outcomes of other tests.

d-disjunct matrices

1. a mathematical model for non-adaptive group testing
2. the property posed can be applied so that each set of at most d positives corresponds to a unique outcome vector.

non-adaptive pooling design

the design of a d -disjunct matrix with various additional considerations

e-error-correcting pooling design:

1. a pooling design is *e-error-correcting* if up to e errors in test outcomes can be tolerated;
2. the test outcome vectors form a binary code of length n and with minimum Hamming distance at least $2e+1$.

Let $D = \{x \mid x \in Z_2{}^n \text{ with weight } d\} \subseteq Z_2{}^n$,
looking for a matrix M of order $t \times n$,

$$x \in D \subseteq Z_2{}^n \text{ (message)} \;\rightarrow\; r(x) = \overline{Mx} \in Z_2{}^t \text{ (encoded message)}$$

$$\rightarrow\; r(x) + e \in Z_2{}^t \text{ (reported message)} \;\rightarrow\; x \text{ (decoded)}$$

such that the minimum distance of the set $\{\overline{Mx} \mid x \in D\} \subseteq Z_2{}^t$ is as large as possible, but at least 1 for error-correcting purpose.

Recently, the notion of non-adaptive pooling designs has been expanded to the case that the units to be identified are not molecules, but subsets of molecules, called *complexes*.

group testing for complexes:

to identify an unknown family $\Im = \{D_1, D_2, ..., D_d\}$ of *k*-subsets of [*t*], called positive *k-complexes*, by performing

1. a series of (0, 1) tests on subsets (pools) of [*t*];
2. a pool is *positive* (1) if it *contains a positive complex completely*; and negative (0) otherwise.

In classical group testing, each member of $\Im$ is a singleton.

2. from *disjunct matrices* to *generalized cover free families*

Definition: *d-disjunct*

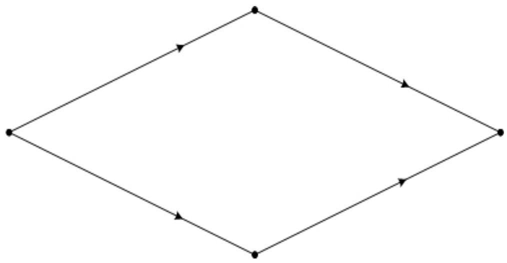
A $(0,1)$ -matrix M of order $t\times n$ is called *d-disjunct* if the set system $\{C_1,...,C_n\}\subseteq 2^{[t]}$ satisfies the condition that $C_j \subsetneq \bigcup_{i\in D} C_i$, i.e., $|C_j - \bigcup_{i\in D} C_i| \geq 1$ whenever $D\subseteq [n]$ with $|D|\leq d$, and $j\notin D$

1. a strategy for a *non-adaptive group testing* which can identify up to d positives:
   Let $C_i\subseteq [t], T_j\subseteq [n]$ be the subsets with the $i-th$ column and the $j-th$ row of M as their characteristic vectors;
2. to identify a positive subset, unknown at the beginning, of the population $[n]$, the tests $T_1,T_2,...,T_t$ over the population $[n]$ are arranged in advance; an outcome vector $x_D=(x_1,x_2,...,x_t)^t$ will be reported after these t tests performed simultaneously, where $x_j=1$ if and only if $T_j\bigcap D$ is nonempty.

Theorem Let $D\in\binom{[n]}{d}$, if $\{C_1,...,C_n\}\subseteq 2^{[t]}$ is d-disjunct, then the dual family $\{T_1,T_2,...,T_t\}\subseteq 2^{[n]}$ satisfies the condition that $\bigcup_{j\notin c(D)} T_j = [n]-D$, and vice versa.

$(1, d; e+1)$
$d^e - disjunct$
D'yachkov 1983, Macula 1998

$(1, d; 1)$
superimposed codes
Kautz, Singleton
1964



$(s, l; e)$
generalized cover families
Stanton, Wei 2004

$(s, l; 1)$
superimposed (s, l) *codes*
D'yachkov, Vilenkin, Macula, Torney 2002

All these structures

d-disjunct matrices: Kautz and Singleton (1964),

(s,l)-*superimposed codes* and *designs*: D'yachkov and Torney *et*. *al*. (2002), and

$(s,l;e)$ - *generalized cover free families*: Stinson, Wei (2004)

can be used in combinatorial group testing algorithms applicable to DNA library screening, and they are therefore called *pooling designs*.

1. for eror-correcting purpose,
   d-disjunct matrices with $(s,l,e) = (1,d;0)$ is generalized to
   d^e - disjunct matrices with $(s,l,e) = (1,d;e+1)$; and

2. for group testing over complexes purpose,
   d-disjunct matrices is generalized to
   (s,l)-superimposed codes with $(s,l,e) = (s,l;0)$;

3. for the capability of error-correcting over complexes,
   d-disjunct matrices is generalized to
   $(s,l)^e$ - *cover free families*:

Definition $(s,l)^e$ - *cover free families* (Stinson and Wei 2004)

Let s, l and e be positive integers, a set system $(X,\mathfrak{I})$ is called a $(s,l)^e$ -*cover-free-family* provided that $|\bigcap_{j\in S} C_j - \bigcup_{i\in L} C_i| \geq e$ for disjoint $S, L \subseteq [n]$ with $|S| \leq s$, and $|L| \leq l$. Less formally, the intersection of any s blocks contains at least e elements not in the union of l other blocks.

Compare the conditons:

1. d -*disjunct*:

   $|C_j - \bigcup_{i\in D} C_i| \geq 1$

2. d^e -*disjunct*: for *error-correcting* purpose

   $|C_j - \bigcup_{i\in D} C_i| \geq e$

3. *superimposed* (s, l)-*code*:

   $|\bigcap_{j\in S} C_j - \bigcup_{i\in L} C_i| \geq 1$ whenever $|S| \leq s$ and $|L| \leq l$;

4. $(s,l)^e$ -*cover free families*:

   $|\bigcap_{j\in S} C_j - \bigcup_{i\in L} C_i| \geq e$

5. $(s,l)^e$ - setwise disjunct matrices:

Two families of disjunct matrices

1. the binary incidence matrix of the system $(\binom{[n]}{d}, \binom{[n]}{k}; \subseteq)$ is *d*- disjunct. (Macula 1996)

2. the binary incidence matrix of order of the system $(\begin{bmatrix} F_q^n \\ d \end{bmatrix}, \begin{bmatrix} F_q^n \\ k \end{bmatrix}; \subseteq)$ is *d*- disjunct.

(Ngo, D.-Z. Du 2003)

In addition to the Boolean algebra $(2^{[n]}, \subseteq)$, Ngo and D.-Z. Du asked the following questions:

1. what are other lattices we can use?
2. what are conditions for which the incidence matrices of some two levels on the lattices for constructing d-disjunct matrices? restrain to lattices with some regularity constraints;
3. derive error correcting capability of the matrices from the lattices ?

Ngo, D.Z. Du pointed out:

”… *this is a young and interesting field with deep connections to coding theory and design theory. We strongly believe that the theory of distance regular graphs, in particular association schemes, should play an important role in improving our pooling designs*.”

A Survey on Combinatorial Group Testing Algorithms with Applications to DNA Library Screening (DIMACS 2000)

3. Pooling spaces and some families of examples

In addition to

the Boolean algebra $(2^{[n]}, \subseteq)$, and

the binary incidence matrix of the system $(\binom{[n]}{d}, \binom{[n]}{k}; \subseteq)$;

Ngo, D.-Z. Du proposed the following questions:

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recall that a ranked poset P is *atomic* if each element x of P is the least upper bound of the subposet $[0, x] \cap P_1$.

Definition (Weng and H 2004)
A *pooling space* is a ranked partially ordered set $P=(X,\leq)$ such that $w^{+}=\{\, y\geq w \mid y\in X\}$ is atomic for all $w\in P$.

Some examples of pooling spaces:

1. ranked semi-lattices such that each interval is atomic,
2. combinatorial geometries.

Some d -disjunct matrices can be derived from pooling spaces:

Theorem (Weng & H 2004) Let P be a pooling space with rank $D\geq 1$ and an integer $l\in[d,D]$.

1. fix $x\in P_D$ and an integer $d\in[1,D]$, let $T\subseteq P_D$ be a subset with $|T|\leq d$ and $x\notin T$, then

   there exists $y\in[0,x]\cap P_d$ such that $\neg(y\leq z)$ for all $z\in T$.

2. each $w\in[y,x]\cap P_l$ satisfies $w\leq x$ and $w>z$ for all $z\in T$.

3. the incidence matrix $M=M(l,D)$ of $(P_l,P_D;\leq)$ is d^e -disjunct, where

$$e=\min\left|\bigcup[y,x]\cap P_l\right|-1$$

   the minimum taken from the pair (x,T) such that $x\in P_D$, $T\subseteq P_D$, $x\notin T$, $|T|\leq d$, and the union taken over all y such that $y\leq x$, $\neg(y\leq z)$ for all $z\in T$.

Optimal error correcting capability derived from
pooling spaces with intervals carrying the structure of *projective geometries*:

Theorem (D'yachkov et. al. 2005)

Let $G_q(n,k,r)$ be the incidence matrix of the incidence structure $(\begin{bmatrix} F^n \\ r \end{bmatrix}, \begin{bmatrix} F^n \\ k \end{bmatrix}; \subseteq)$ where

$1 \le r < k \le n$ with $k - r \ge 2$, then

1. $G_q(n,k,r)$ is d^z-disjunct for $1 \le d \le \dfrac{q(q^{k-1}-1)}{q^{k-r}-1}$, and

2. $G_q(n,k,r)$ is d^z-disjunct but not d^{z+1}-disjunct if $1 \le d \le q+1$, and

$$z = q^{k-r}\begin{bmatrix} k-1 \\ r-1 \end{bmatrix}_q - (d-1)q^{k-r-1}\begin{bmatrix} k-2 \\ r-1 \end{bmatrix}_q .$$

Theorem

Let P be a pooling space of rank D such that each interval of rank i in P is isomorphic to the projective geometry $PG(i,q)$, and let $M(d,k)$ be the incidence matrix of the incidence structure $(P_d, P_k; \le)$ for $1 \le d < k \le D$ with $k - d \ge 2$. Then $M(d,k)$ is s^z-disjunct for

$1 \le s \le \dfrac{q(q^{k-1}-1)}{q^{k-r}-1}$ and $z = q^{k-r}\begin{bmatrix} k-1 \\ r-1 \end{bmatrix}_q - (d-1)q^{k-r-1}\begin{bmatrix} k-2 \\ r-1 \end{bmatrix}_q$.

In addition to $(\begin{bmatrix} F^m \\ r \end{bmatrix}, \begin{bmatrix} F^m \\ k \end{bmatrix}; \subseteq)$, its substructure the *attenuated space* of rank D is another example:

Theorem

Let $M(D,k,d)$ be the incidence matrix of $(P_k, P_d, \leq)$ of the pooling space associated with the *attenuated space* of rank *D*, and fix integers $1 \leq r < k \leq D$ with $k - r \geq 2$.

Then $M(D,k,d)$ is s^z - disjunct for $1 \leq s \leq \dfrac{q(q^{k-1}-1)}{q^{k-d}-1}$ and

$$z = q^{k-d} \begin{bmatrix} k-1 \\ d-1 \end{bmatrix}_q - (s-1)q^{k-d-1} \begin{bmatrix} k-2 \\ d-1 \end{bmatrix}_q .$$

4. Some connections with the Johnson graphs and the Grassmann graphs

Some d^e -disjunct matrices for certain values of *e*:

1. the incidence matrix $J(n,d,k)$ of the system $(\binom{[n]}{d},\binom{[n]}{k};\subseteq)$

(*J* is for *Johnson Schemes*, by Macula 1996)

2. the incidence matrix $G_q(n,d,k)$ of the system $(\binom{GF(q)^n}{d},\binom{GF(q)^n}{k};\subseteq)$

(*G* is for *Grassmann Schemes*, by Du 2003)

3. the incidence matrix $M(2n,d,k)$ of the system $(d-\text{matchings}, k-\text{matchings};\subseteq)$ over K_{2n}

(*M* is for *matchings*, by Du 2003)

Theorem (Du, Ngo 2003)

For $m \geq k > d \geq 1$,

1. $M(m,k,d)$ is a *d*-disjunct matrix.

2. $M(m,m,d)$ is *d*-error-detecting and $\lceil d/2 \rceil$-error-correcting.

Question:

What is the *q*-analogue of the incidence matrix $M(2n,d,k)$ of the matchings over K_{2n} ?

Question:

What is the q-analogue of the incidence matrix $M(2n,d,k)$ of the matchings over K_{2n} ?
With an interpretation of

matchings of K_{2m} as

2-cliques of the Johnson graph $J(2m,2)$,

this gives a q-analogue of the pooling designs defined over matchings of K_{2m} .

Based on the inclusion matrices of t -cliques with various sizes of the Johnson graphs $J(n,t)$ and the Grassmann graphs $J_q(n,t)$ respectively, two families of error-correcting pooling designs are given. These pooling designs have the same capability of error-detecting and error-correcting as Ngo and Du's, however the test to item ratio of ours is much smaller.

Definition

1. The **Johnson graph** $J(n,t)$ is the graph defined on the family of all t-elements subsets of the set $\{1, 2, ..., n\}$ such that A, B are adjacent if $|A \cap B| = t-1$
2. The **Grassmann graph** $J_q(n,t)$ is the graph defined on the family of all t-dimensional subspaces of a vector space of dimension n over F_q such that A, B are adjacent if the dimension of $A \cap B$ is t-1.

Both Johnson graphs and Grassmann graph are **distance-regular graphs**.

A clique (1-clique) of $J(n,2)$ is a subfamily of 2-elements subsets such that $|A \bigcap B| = 1$ for any two distinct *A*, *B*. An *d*-matching of K_{2m} is simply a family of size *d* of 2 -subsets of [*n*] which are pairwise disjoint, a 2-clique of $J_q(2m,2)$ of size *l* is an *q*-analogue of an *l*-matching of K_{2m}.

Definition

1. A *t*-clique of $J(n,t)$ with size *l* is a subfamily $\{A_1, ..., A_l\}$ of *t*-elements subets which pairwise intersect trivially. A *t*-clique of $J_q(n,t)$ with size *l* is a subfamily $\{A_1, ..., A_l\}$ of t-dimensional subspaces which pairwise intersect trivially.

2. A family of *k*-subsets in [*n*] with $|K \cap K'| \leq k - t$ for all *K* and K' in $\mathcal{K}$ is called a {1,2,…,t}-clique of $J(n,k)$.

Theorem

Let $m \geq k > r > d \geq 1$, the matrix $MJ(m,t,k,r)$ is d^e-disjunct, but not d^{e+1}- disjunct where $e = \binom{k-d}{r-d} - 1$.

Theorem

Let $m \geq k > d \geq 1$, then the matrix $MG_q(m,t,k,r)$ is d^e-disjunct, but not d^{e+1}-disjunct, where $\begin{bmatrix} k-d \\ r-d \end{bmatrix} - 1$.

Both values of e are optimal if $m > k$.

Theorem (Macula 1997) For $1 \le d \le k \le n$ and $1 \le r \le k$, let $\mathcal{K}$ be a family of *k*-subsets of [*n*] with the minimum Hamming distance $d_H(K)$ between any pair of *k*-sets in $\mathcal{K}$ is at least 2*r*, then

1. $J(n, d, \mathcal{K})$ is $d^{\alpha_d - 1}$- disjunct where $\alpha_d = \min(r^d, k-d)$.(Theorem 2).

2. $J(n,d,k,K,r)$ is s^e-disjunct if $1 \le s \le p$, where

$$p = \left\lceil \left(\binom{k}{d} - \binom{k-r}{d} \right) \left(\binom{k-r}{d} - \binom{k-2r}{d} \right)^{-1} \right\rceil \text{ and}$$

$$e = \binom{k}{d} - \binom{k-r}{d} - (s-1)\left(\binom{k-r}{d} - \binom{k-2r}{d} \right) - 1 .$$

The following lemma is used in the proof of the following theorem.

Remarks

1. Let $\mathcal{K}$ be a family of *k*-subsets in [*n*] with $|K \cap K'| \le k - t$ for all K and K' in $\mathcal{K}$. Let $d \ge 1$ with $t \ge 1 + t/(k-d)$ and set $\alpha_d = \min(t^d, k-d)$. Then given d+1 *k*-sets $\{K_i\}_{i=0}^{d} \subset \mathcal{K}$, there are α_d *d*-sets $\{D_j\}_{j=1}^{\alpha_d}$ in [*n*] such that each D_j is contained in K_0 and no D_j is connected in K_i for $1 \le i \le d$.

2. Find examples of $\Gamma \subseteq \binom{[n]}{k}$ with $d_H(\Gamma) \ge 2r$? study their properties?

5. Pooling designs on complexes

The notion of pooling designs on complexes can be traced back to Torney in 1999, called *sets pooling designs*, and was carried out by D'yachkov, Vilenkin, Macula, and Torney (2004) as follows:

for positive integers s, l and t such that $s + l \leq t$, let

$$\wp(s,l,t) \ = \text{the family of all antichains} \{P_1, P_2, ..., P_k\}$$

$$\text{with } P_i \subseteq [t] \ \text{ and } \ |P_i| \leq l \text{ for each } \ i \leq k \leq s .$$

Definition (D'yachkov, Vilenkin, Macula, and Torney 2004)

A binary matrix M of order $N \times t$ is called

1. a *superimposed* (s,l) *-code* if, for any two disjoint subsets S, L of $[t]$ with $|S| = s$ and $|L| = l$, there exists a row with entry 1 over L and 0 over S.

2. a *superimposed* (s,l) *- design* if $\bigcup_{P_i \in \wp} \left(\bigcap_{j \in P_i} C_j \right) \neq \bigcup_{P_i' \in \wp'} \left(\bigcap_{j \in P_i'} C_j \right)$ for distinct

$$\wp = \{P_1, P_2, ..., P_k\} , \quad \wp' = \{P_1', P_2', ..., P_h'\} \in \wp(s,l,t) .$$

Note that the point-block incidence matrix of a $(l,s;1)$-*cover-free-family* is indeed a superimposed (s,l)-code. As a common generalization of d^e-disjunct matrices and $(s,l)^e$-*cover-free-families*, the notion of $(s,l)^e$- *setwise disjunct* matrices is introduced for pooling designs on complexes.

Definition: For positive integers s,l with $s+l \le t$, a binary matrix M of order $N \times t$ is called an $(s,l)^e$- *setwise disjunct* matrix if

$$\left| \bigcap_{i \in A} C_i - \bigcup_{P_i \in \wp} \left(\bigcap_{j \in P_i} C_j \right) \right| \ge e$$

for any antichain $\wp = \{P_1, P_2, ..., P_k\} \in \wp(s,l,t)$, and for any $A \subseteq [t]$ with $|A| \le l$ and $A \notin \wp$.

Theorem 2 The point-block incidence matrix *M* of an $(l,s)^e$ - cover free family $\{C_1, C_2, ..., C_t\}$ is an $(s,l)^e$ - setwise disjunct matrix.

A $(s,l)^e$ - *setwise disjunct* matrices *M* can be used for a pooling design on complexes:

1. the columns of *M* be identified with the set of samples, and its rows are identified with pools for testing such that $M(i, j) = 1$ if the *j*-th sample is included in the *i*-th pool.

2. the set of samples $[t] = \{1, 2, ..., t\}$ with a (unknown and to be identified) positive complex $\wp = \{P_1, P_2, ..., P_k\} \subseteq \wp([t])$, each test checks whether a pool contains at least one positive set $P_i \in \wp$ completely.

3. the outcome vector

   $$o(\wp) = o(\wp, M) = \text{the characteristic vector of the set } \bigcup_{P_i \in \wp}\left(\bigcap_{j \in P_i} C_j\right)$$

   is reported after the test;

4. if the report $o(\wp) + \varepsilon$ with an error vector ε is received, the error can be detected whenever $\text{weight}(\varepsilon) \le e$, or even the errors can be corrected whenever $\text{weight}(\varepsilon) \le \left\lfloor \frac{e-1}{2} \right\rfloor$.

In case $l = 1$, then each $P_i \in \wp$ is reduced to a singleton, and it then reduces to d^e - disjunct matrices whenever $k = d$.

6. Decoding

The methodology of Kautz-Singleton has been generalized to decoding methods for pooling designs based on d^e - disjunct matrices and on $(s,l)^e$ - disjunct matrices as well.

6-1 Observation

Let $D \in \binom{[n]}{d}$, if $\{C_1, ..., C_n\} \subseteq 2^{[t]}$ is *d*-disijunct, then the dual family $\{T_1, T_2, ..., T_t\} \subseteq 2^{[n]}$ satisfies the condition that $\bigcup_{j \notin c(D)} T_j = [n] - D$, and vice versa.

6-2. A decoding algorithm for d^e disjunct matrices

Theorem (Weng and H 2003) Let M be a d^e - disjunct matrix of order $t \times n$,

1. $d_H(M(P_1), M(P_2)) \geq e$ for any two distinct (positive) subsets $P_1, P_2 \subseteq \{1, 2, \ldots, n\}$ with $|P_i| \leq d$ each.

2. Let $P \subseteq [n]$ with $|P| \leq d$ and $U \subseteq [t]$, and let $T = \{C_j \,\big|\, |C_j - \chi_U| \leq \left\lfloor \frac{e-1}{2} \right\rfloor\}$,

   a. if $d_H(M(P), \chi_U) \leq \left\lfloor \frac{e-1}{2} \right\rfloor$, then $T = P$.

   b. if $d_H(M(P), \chi_U) \leq e$ and $|T| \leq d$, then $M(P) = \chi_U$ if and only if $M(T) = \chi_U$

The above theorem provides a decoding algorithm.

6-3. A decoding algorithm for $(s,l)^e$ - setwise disjunct matrices

Let χ_A with $A \subseteq [N]$ be the output vector for the group testing over the (to be identified) positive family $\wp = \{P_1, ..., P_k\}$, the following theorem provides an decoding algorithm over the $(s,l)^e$ - setwise disjunct matrix *M*.

For an $(s,l)^e$ - setwise disjunct matrix *M*, we are interested to know the minimum distance, i.e., the minimum of the set { $d_H(o(\wp), o(\wp')) \mid \wp, \wp' \in \wp(s,l,t)$ }.

Theorem An $(s,l)^e$ - setwise disjunct matrix is a superimposed (s,l)-design with the minimum distance at least $2e$.

Theorem Let $A \subseteq [N]$, and let

$$\wp_A = \left\{ Z \,\middle|\, |Z| \le l, \text{ and } |\bigcap_{j \in Z} C_j - \chi_A| \le \left\lfloor \frac{e-1}{2} \right\rfloor \right\}.$$

Then the following hold:

1. If $d_H(o(\wp), \chi_A) \le \left\lfloor \frac{e-1}{2} \right\rfloor$, then $\wp = \wp_A$.

2. If $d_H(o(\wp), \chi_A) \le e-1$ and $\left| \bigcup_{B \in \wp_A} B \right| \le s$, then $o(\wp) = \chi_A$ if and only if $o(\wp_A) = \chi_A$.

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