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Improved upper bounds for nearly antipodal chromatic number of paths

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1 Basic concepts

Nearly antipodal coloring is a special setting of the radio k-colorings.

Radio k-colorings are generalizations of ordinary colorings of graphs, which were inspired by (FM Radio) Channel Assignments Problem (see [5, 6]) and introduced by G. Chartrand, D. Erwan, F. Harary and P. Zhang [1].

For a connected graph G of order n and diameter d and a integer k with $1 \le k \le d$, a radio k-coloring of G is a function

$$c : V(G) \to \mathbf{N},$$

such that

$$d(u, v) + |c(u) - c(v)| \ge k + 1$$

for every pair u and v of distinct vertices of G, where $d(u, v)$ denotes the distance between u and v (the length of a shortest $u - v$ path) in G.

Clearly, radio 1-colorings and ordinary colorings are synonymous.

The *value* $\mathrm{rc}_k(c)$ of a radio k-coloring c of G is the maximum color assigned to a vertex of G; while the *radio k-chromatic number* $\mathrm{rc}_k(G)$ of G is $\min\{\mathrm{rc}_k(c)\}$ taken over all k-colorings of G.

In particular, radio d-coloring is referred to as *radio labeling*, and the *radio d-chromatic number* is called the *radio number*.

Note that in a radio $(d-1)$-coloring of G, it is possible for two vertices u and v to be colored the same, but only if they are antipodal, that is, $d(u, v) =$diam $(G) = d$. For this reason, a radio $(d-1)$-coloring was also referred to as a *radio antipodal coloring* or, more simply, as an *antipodal coloring*. And the *radio $(d-1)$-chromatic number* is called the *antipodal chromatic number*, denoted by $\mathrm{ac}(G)$.

Radio k-coloring and radio d-coloring (radio labeling) of graphs were studied in [1, 2]. Radio antipodal coloring of paths was studied in [3, 4].

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Furthermore, G. Chartrand, L. Nebeský and P. Zhang gave the concepts of *nearly antipodal colorings* in [4].

For a connected graph G of diameter d, a nearly antipodal coloring of G is a function

$$c : V(G) \to \mathbf{N},$$

such that

$$d(u,v) + |c(u) - c(v)| \geq d - 1$$

for every two distinct vertices u and v of G. The *value* $\mathrm{rc}'(c)$ of a nearly antipodal coloring c of G is the maximum color assigned to a vertex of G. The *nearly antipodal chromatic number* $\mathrm{rc}'(G)$ of G is $\min\{\mathrm{rc}'(c)\}$ taken over all nearly antipodal colorings of G.

In fact, for $d \geq 3$, a nearly antipodal coloring is a radio $(d-2)$-coloring.

Clearly, if G is a connected graph of diameter 1 or 2, then $\text{ac}'(G) = 1$; while if $\text{diam}(G) = 3$, then $\text{ac}'(G)$ is the chromatic number of G. Thus nearly antipodal colorings are most interesting for connected graphs of diameter 4 or more. For this reason, the nearly antipodal chromatic number of paths P_n were investigated in [4] by G. Chartrand, L. Nebeský and P. Zhang. And they showed that

$$\text{ac}'(P_5) = 5, \text{ac}'(P_6) = 7, \text{ac}'(P_7) = 11 \text{ and } \text{ac}'(P_8) = 16.$$

Moreover, they presented an upper bound for the nearly antipodal chromatic number of paths P_n for every positive integer n as follows.

Theorem 1([4]). If n is a path of order $n \geq 1$, $ac'(P_n) \leq \binom{n-2}{2} + 2$.

2 Main results

In this paper we will present an improved version for Theorem 1. We will show that

Theorem 2. 1. If n is even and $n \geq 10$, then

$$ac'(P_n) \leq \binom{n-2}{2} - \frac{n}{2} - \lfloor \frac{10}{n} \rfloor + 7;$$

2. If n is odd and $n \geq 13$, then

$$ac'(P_n) \leq \binom{n-2}{2} - \frac{n-1}{2} - \lfloor \frac{13}{n} \rfloor + 8.$$

Clearly, it holds that $-\frac{n}{2} - \lfloor \frac{10}{n} \rfloor + 7 \leq 1$ for all even integers $n \geq 10$, and $-\frac{n-1}{2} - \lfloor \frac{13}{n} \rfloor + 8 \leq 1$ for all odd integers $n \geq 13$. Thus, for all even integers $n \geq 10$ and all odd integers $n \geq 13$, Theorem 1 improves the upper bounds of $\mathrm{ac}'(P_n)$.

3 Sketch of the proof

1. *n is even and $n \geq 10$.*

Firstly, we let $n \geq 12$, note that $-\lfloor \frac{10}{n} \rfloor = 0$, it suffices to show that $\mathrm{ac}'(P_n) \leq \binom{n-2}{2} - \frac{n}{2} + 7$. Write $n = 2k = 10 + 2(4p+q)$, where $p \in \{0, 1, 2, \ldots\}$ and $q \in \{1, 2, 3, 4\}$. Then we have that $k = 5 + (4p+q)$ and $d - 1 = \mathrm{diam}(P_n) - 1 = 2k - 2$.

Denoted the vertices of P_n by (see Figure 1)

$$V_1 = \{x_1, x_2; y_1, y_2; x'_1, x'_2, x'_3; y'_1, y'_2, y'_3\},$$
$$V_2 = \{v_1, u_2, v_3, u_4, \ldots, v_{2p-1}, u_{2p}; v'_1, v'_2, \ldots, v'_{2p-1}, v'_{2p}; u'_1, u'_2, \ldots, u'_{2p-1}, u'_{2p}\},$$
$$V_3 = \{w_1, w_2, \ldots, w_q; z_1, z_2, \ldots, z_q; v_{2p}, u_{2p-1}, \ldots, v_4, u_3, v_2, u_1\}.$$

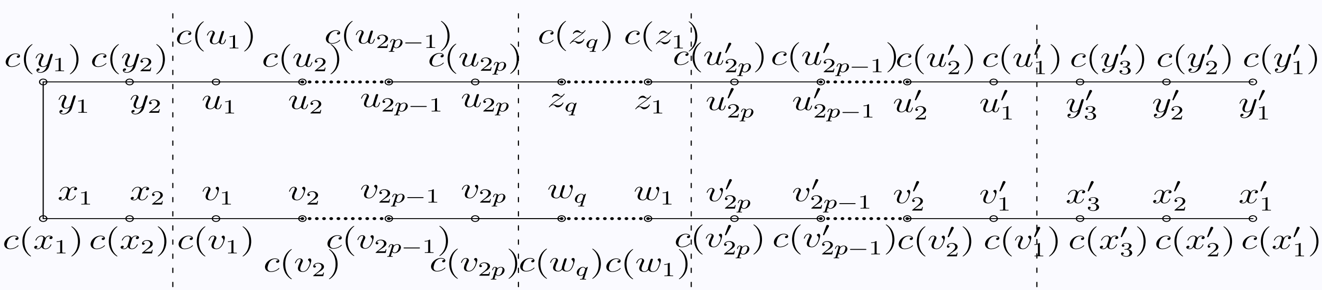

Figure 1: A nearly antipodal coloring for $P_n(n = 2k \geq 10)$

In the following we will present a coloring c for P_n by three steps, such that

$$d(u,v) + |c(u) - c(v)| \geq d - 1 = 2k - 2 \qquad (1)$$

holds for all distinct vertices $u, v \in V_1 \cup V_2 \cup V_3 = V(P_n)$, and $\mathrm{ac}'(c) = \binom{n-2}{2} - \frac{n}{2} + 7$.

Step 1. *Color the vertices in* V_1 (see Figure 1).

Step 2. *Color the vertices in* V_2 (see Figure 1).

Step 3. *Color the vertices in* V_3 (see Figure 1).

Step 3.1. *Color the vertices in* $\{w_1, \ldots, w_q; z_1, \ldots, z_q\}$ (see Figure 1).

Step 3.2. Color the vertices in $\{v_{2p}, u_{2p-1}, \ldots, v_4, u_3, v_2, u_1\}$ (see Figure 1).

From the above three steps, we can establish four Claims and show that $\mathrm{ac}'(P_n) \leq \mathrm{ac}'(c) = \binom{n-2}{2} - \frac{n}{2} + 7$ for all even integers $n \geq 12$.

Secondly, for $n = 10$, in the above proof we take $p = 0$ and $q = 0$. Namely, $V_2 = V_3 = \Phi$, $V(P_{10}) = V_1 = \{x'_1, x'_2, x'_3; x_2, x_1; y_1, y_2; y'_3, y'_2, y'_1\}$ (also see Figure 1 and let $p = q = 0$). Then coloring $c|_{v\in V_1}(v)$ is a nearly antipodal coloring for P_{10}. Thus by Claim 2, $\mathrm{ac}'(P_{10}) \leq \mathrm{ac}'(c|_{v\in V_1}) = \max_{v\in V_1} c(v) = c(y_2) = (6k-1)|_{k=5} = 29 = \binom{10-2}{2} + 1$.

Since $-\lfloor\frac{10}{n}\rfloor = -1$ for $n = 10$, it follows that $\mathrm{ac}'(P_{10}) \leq \mathrm{ac}'(c|_{v\in V_1}) = \binom{10-2}{2} = \binom{10-2}{2} - \frac{10}{2} - \lfloor\frac{10}{10}\rfloor + 7$.

Combine the above, we complete the proof of assertion 1 in Theorem 2.

2. *n is odd and $n \geq 13$.*

Firstly, we let $n \geq 15$, note that $-\lfloor \frac{13}{n} \rfloor = 0$, it suffices to show that $\mathrm{ac}'(P_n) \leq \binom{n-2}{2} - \frac{n}{2} + 8$. Write $n = 2k+1 = 13 + 2(4p+q)$, where $p \in \{0, 1, 2, \ldots\}$ and $q \in \{1, 2, 3, 4\}$. Then we have that $k = 6 + (4p+q)$ and $d-1 = \mathrm{diam}(P_n) - 1 = 2k-1$.

Denoted the vertices of P_n by (see Figure 2)

$$V_1 = \{x_0; x_1, x_2; y_1, y_2; x'_1, x'_2, x'_3, x'_4; y'_1, y'_2, y'_3, y'_4\},$$
$$V_2 = \{v_1, u_2, v_3, u_4, \ldots, v_{2p-1}, u_{2p}; v'_1, v'_2, \ldots, v'_{2p-1}, v'_{2p}; u'_1, u'_2, \ldots, u'_{2p-1}, u'_{2p}\},$$
$$V_3 = \{w_1, w_2, \ldots, w_q; z_1, z_2, \ldots, z_q; v_{2p}, u_{2p-1}, \ldots, v_4, u_3, v_2, u_1\}.$$

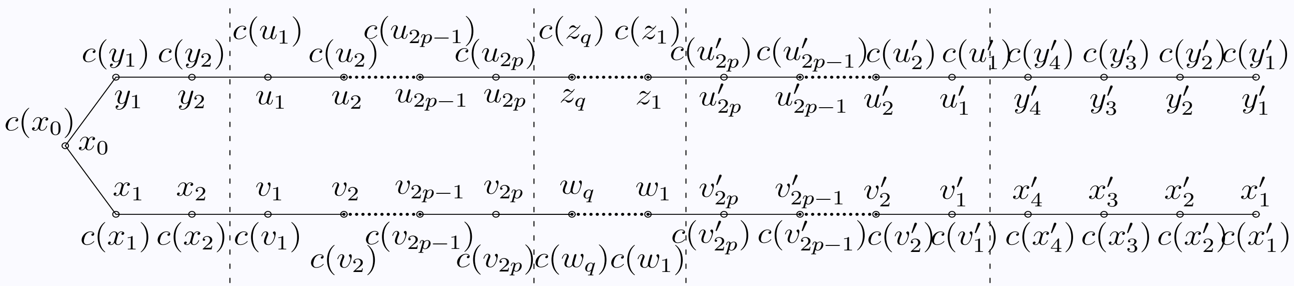

Figure 2: A nearly antipodal coloring for $P_n (n = 2k+1 \geq 13)$

Similar to the proof of the case n being even, we can show that the assertion 2 in Theorem 2 holds.

4 Some examples

Example 1. A nearly antipodal coloring c for P_{10} with $ac'(c) = \binom{10-2}{2} - \frac{10}{2} - \lfloor \frac{10}{10} \rfloor + 7 = \binom{10-2}{2} + 1 = 29$ (see Figure 3).

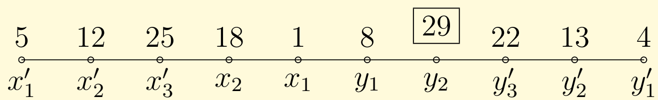

Figure 3: A nearly antipodal coloring for P_{10}

Example 2. A nearly antipodal coloring c for P_{32} with $ac'(c) = \binom{32-2}{2} - \frac{32}{2} + 7 = \binom{32-2}{2} - 9 = 426$ (see Figure 4).

Here $n = 2k = 10 + 2(4p + q) = 32$, then $k = 16$, $p = 2$ and $q = 3$.

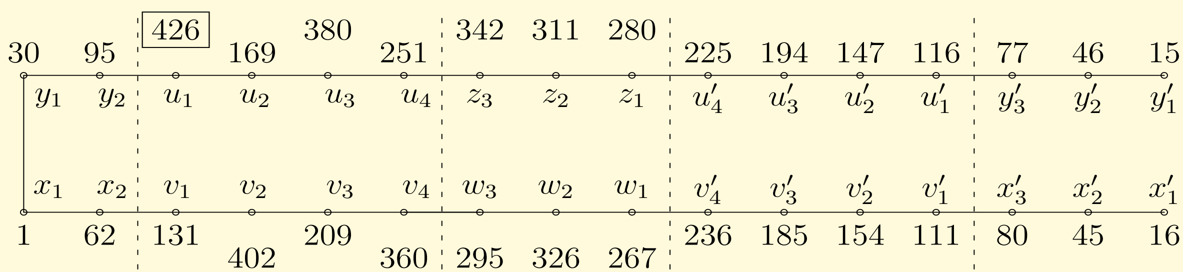

Figure 4: A nearly antipodal coloring for P_{32}

Example 3. A nearly antipodal coloring c for P_{13} with $ac'(c) = \binom{13-2}{2} - \frac{13-1}{2} - \lfloor\frac{13}{13}\rfloor + 8 = \binom{13-2}{2} + 1 = 56$ (see Figure 5).

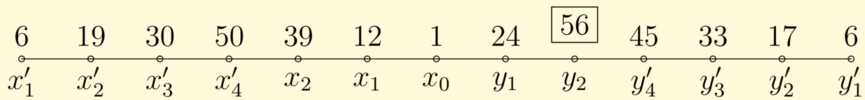

Figure 5: A nearly antipodal coloring for P_{13}

Example 4. A nearly antipodal coloring c for P_{33} with $ac'(c) = \binom{33-2}{2} - \frac{33-1}{2} + 8 = \binom{33-2}{2} - 8 = 457$ (see Figure 6).

Here $n = 2k+1 = 13 + 2(4p+q) = 33$, then $k = 16$, $p = 2$ and $q = 2$.

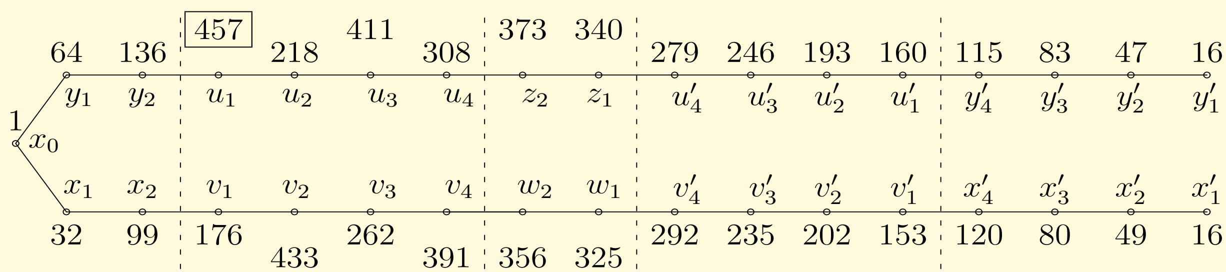

Figure 6: A nearly antipodal coloring for P_{33}

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Thank You!