Factor-Critical Property in 3-Dominating-Critical Graphs

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Definitions and Notations

Dominating set: $S \subseteq V(G)$, if every vertex of G either belongs to S or is adjacent to a vertex of S.

Dominating number: The cardinality of a smallest dominating set, denoted by $\gamma(G)$.

A graph G is said to be *γ-vertex-critical* if $\gamma(G - v) < \gamma(G)$, for every vertex v in G.

Example

3-vertex-critical

Definitions and Notations

A matching is *perfect* if it is incident with every vertex of G.
near-perfect matching: if $\exists$ a $v \in V(G)$, s.t. $G - v$ has a perfect matching.
k-factor-critical: If $G - S$ has a perfect matching, for every $S \subset V(G)$ with $|S| = k$.
For $k = 1, 2$, G is said to be factor-critical, bicritical, respectively.

Definitions and Notations

G is H-free if G has no induceed subgraph isomorphic to H.

D_v: a minimum dominating set of $G - v$.

Some known results

NOTE: The only 1-vertex-critical graph is K_1 (a single vertex).

Theorem (*Brigham, Chinn and Dutton, 1988*) *A graph is 2-vertex-critical if and only if it is obtained from the complete graphs K_{2n} by removing a perfect matching.*

Theorem (*Fulman, Hanson and MacGillivray, 1995*) *The diameter d of a γ-vertex-critical graph G satisfies $d \leq 2(\gamma - 1)$ for $\gamma \geq 2$.*

Some known results

Theorem (*Ananchuen and Plummer, 2005*) *If G is $K_{1,5}$-free 3-vertex-critical of even order, then G has a perfect matching.*

Conjecture (*Ananchuen and Plummer, 2005*) *If G is a 3-vertex-critical graph of even order and $K_{1,7}$-free, then G contains a perfect matching.*

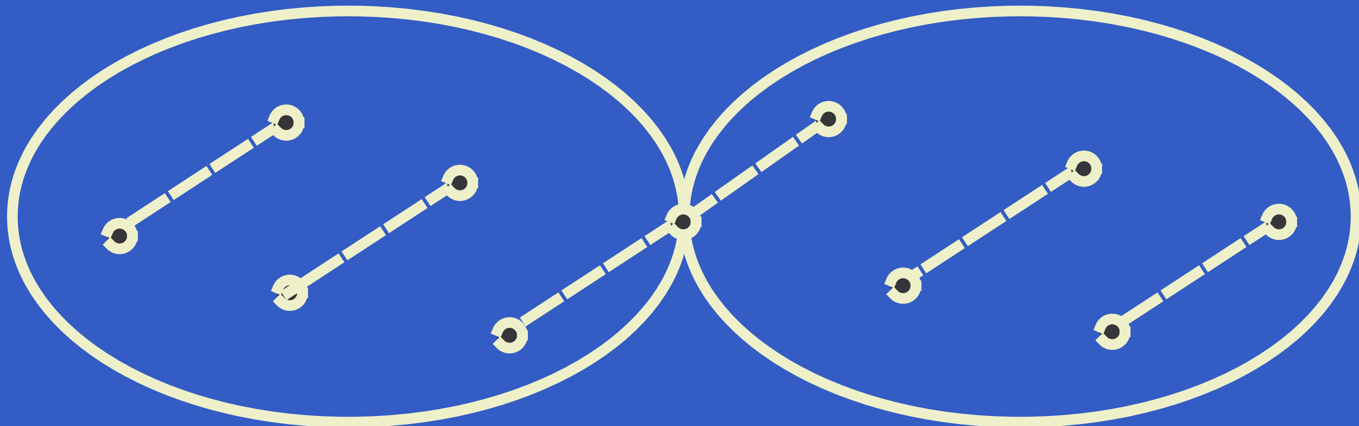
Some known results

Theorem (*Ananchuen and Plummer, 2006*) *Suppose G is a $K_{1,5}$-free 3-vertex-critical graph of odd order at least 11 with $\delta(G) > 0$. Then G contains a near-perfect matching.*

Theorem (*Ananchuen and Plummer, 2006*) *If G is a $K_{1,4}$-free 3-vertex-critical graph of odd order with $\delta(G) \geq 3$, then G is factor-critical.*

Note

NOTE: The hypothesis that the graph is $K_{1,4}$-free cannot be weakened $K_{1,5}$-free.



Conjecture

Conjecture (*Ananchuen and Plummer, 2006*) *If G is a $K_{1,5}$-free 3-vertex-critical 2-connected graph of odd order with $\delta(G) \geq 3$, then G is factor-critical.*

NOTE: This conjecture is not true, we prove this conjecture is true except two counterexamples.

Two counterexamples

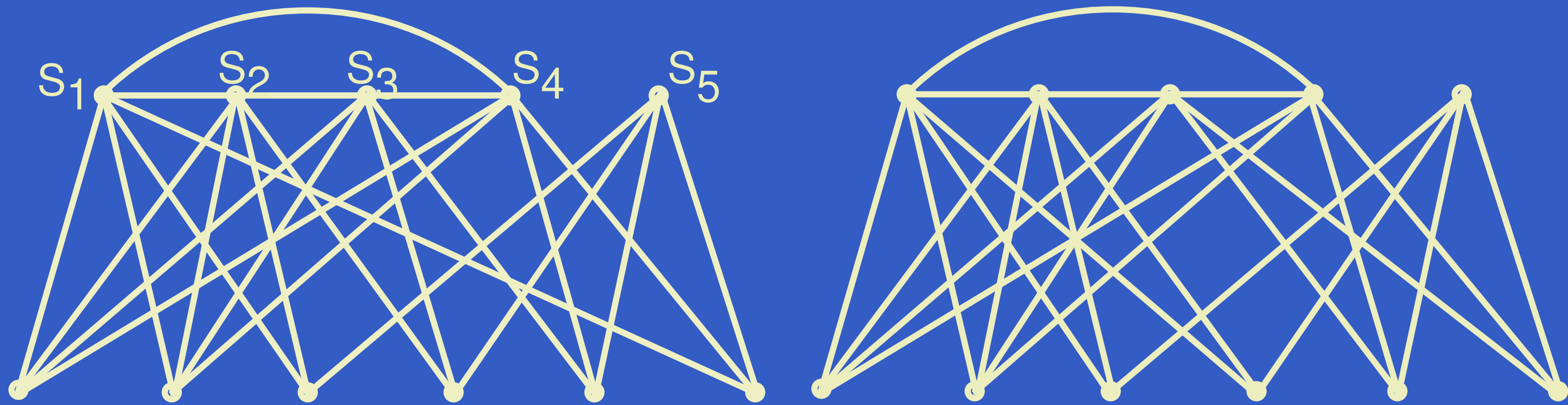


Figure 1: The graphs G_1 and G_2.

Facts

Facts: If G is 3-vertex-critical, then the following hold:

(1) For every vertex v of G, $|D_v| = 2$.

(2) If $D_v = \{x, y\}$, then x and y are not adjacent to v.

(3) For every pair of distinct vertices v and w, $D_v \neq D_w$.

Main Result

Theorem (*Wang and Yu, 2006*) *If G is a $K_{1,5}$-free 3-vertex-critical 2-connected graph of odd order with $\delta(G) \geq 3$ except the graphs G_1 and G_2 shown in Figure 2, then G is factor-critical.*

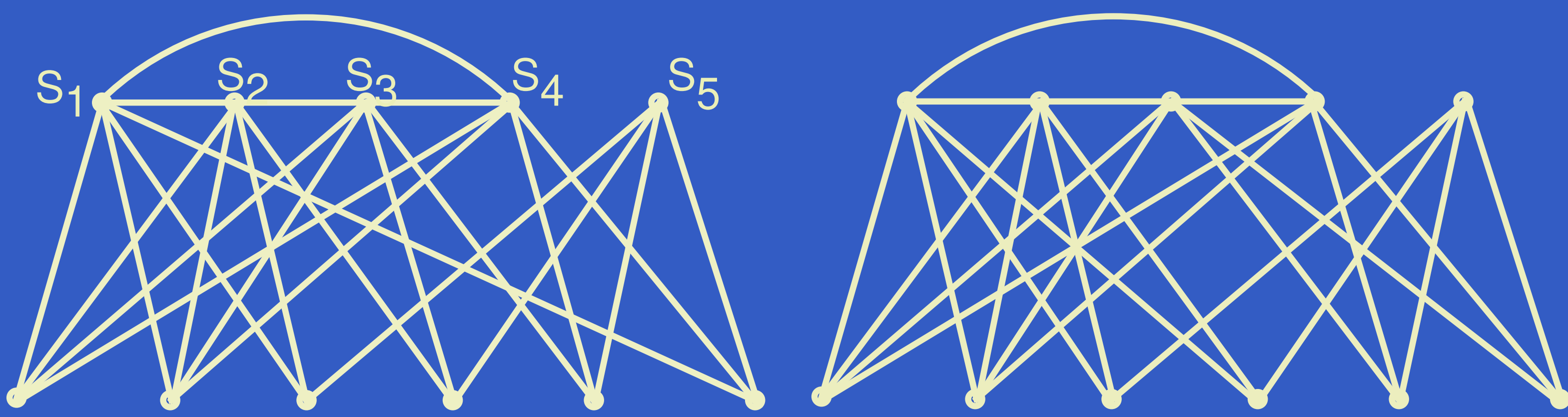


Figure 2: The graphs G_1 and G_2.

Case : $|S| = 5$

- For every vertex $u \in S$, $D_u \subseteq S$.
- $G[S]$ is a 5-cycle or a union of a 4-cycle and an isolated vertex.
- there must exist some $x \in V(G) - S$ such that $D_x \nsubseteq S$
- $G - S$ has 6 odd components and no even components.

Case: $|S| = 5$

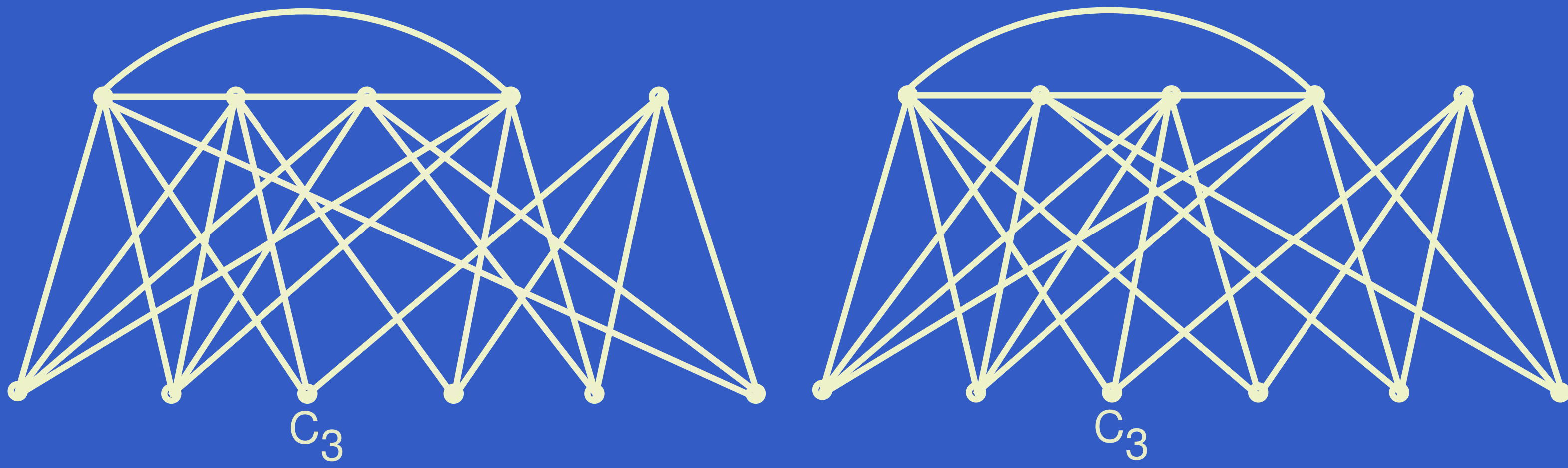


Figure 3: The graphs G_3 and G_4.

THANK YOU FOR YOUR ATTENTION